

METAPHOR AND MATHEMATICS

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ABSTRACT

Traditionally, mathematics and metaphor have been thought of as disparate: the former rigorous, objective, universal, eternal, and fundamental; the latter imprecise, derivative, nearly — if not patently — false, and therefore of merely aesthetic value, at best. A growing amount of contemporary scholarship argues that both of these characterizations are flawed. This dissertation shows that there are important connexions between mathematics and metaphor that benefit our understanding of both. A historically structured overview of traditional theories of metaphor reveals it to be a notion that is complicated, controversial, and inadequately understood; this motivates a non-traditional approach. Paradigmatically shifting the locus of metaphor from the linguistic to the conceptual — as George Lakoff, Mark Johnson, and many other contemporary metaphor scholars do — overcomes problems plaguing traditional theories and promisingly advances our understanding of both metaphor and of concepts. It is argued that conceptual metaphor plays a key role in explaining how mathematics is grounded, and simultaneously provides a mechanism for reconciling and integrating the strengths of traditional theories of mathematics usually understood as mutually incompatible. Conversely, it is shown that metaphor can be usefully and consistently understood in terms of mathematics. However, instead of developing a rigorous mathematical model of metaphor, the unorthodox approach of applying mathematical concepts metaphorically is defended.

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*To Thora, point of my compass, patiently steadfast and
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LIST OF ABBREVIATIONS

BCE	Before the Common Era
CE	Common Era
CMT	Conceptual Metaphor Theory
OED	Oxford English Dictionary
UP	University Press

CHAPTER 1

INTRODUCTION

Mathematics is the classification and study of all possible patterns.

Pattern is here used...in a very wide sense, to cover almost
*any kind of regularity that can be recognized by the mind.*¹

— W.W. Sawyer

What cognitive capabilities underlie our fundamental human achievements? Although a complete answer remains elusive, one basic component is a special kind of symbolic activity — the ability to pick out patterns, to identify recurrences of these patterns despite variation in the elements that compose them, to form concepts that abstract and reify these patterns, and to express these concepts in language. Analogy, in its most general sense, is this ability to think about relational patterns. As Douglas Hofstadter argues...

analogy lies at the core of human cognition.²

— Holyoak, Gentner, and Kokinov

In the Western tradition, mathematics and metaphor have long been thought of as inhabiting opposite ends of the intellectual spectrum. Mathematics is generally considered the paradigm case of rigor and objectivity. Mathematical theorems supported by valid proofs seem to express truths that are exact, eternal, and evident. The precision and certainty afforded by mathematical techniques underpin the extraordinary success of the quantitative sciences. Even though many people find the practice of mathematics difficult and unenjoyable, most nonetheless acknowledge the contribution mathematics makes to the flourishing of our species. Metaphor is generally not regarded so highly. At best, tradition regards it as a convenient linguistic device for communicating subjective experiences (as in poetry) and as a temporary measure when precise, literal language does not yet exist. At worst, metaphor

¹W.W. Sawyer, *Prelude to Mathematics* (Harmondsworth, Middlesex: Penguin, 1955), 12; emphasis his.

²Keith Holyoak, Dedre Gentner, and Boicho Kokinov, “Introduction: The Place of Analogy in Cognition,” *The Analogical Mind: Perspectives from Cognitive Science*, Ed. Gentner, Holyoak, and Kokinov (Cambridge: MIT Press, 2001), 2.

is considered an unnecessary and avoidable figurative impediment to clear communication, a mere step away from outright prevarication. Thus, metaphor has often been conceived as antithetical to but also disparate from mathematics.

There are a variety of reasons to question this traditional view. For one, mathematics and metaphor do not seem as disparate as suggested above. In my decades of experience as a mathematics student, educator, and researcher, I have observed the use of metaphor and analogy at every level of mathematical practice, from young novices learning to add to conference presentations by Fields Medallists. Three specific examples will help substantiate this observation. First, when children are first learning addition their teachers draw a comparison between the addition of numbers and the act of combining collections of physical objects. Second, in order to help students understand the somewhat difficult notion of a mathematical function, teachers and professors often describe functions metaphorically as machines that take in numerical inputs and perform various manipulations and operations upon them to yield numerical outputs. Third, some mathematicians (myself included) use analogy in trying to understand multidimensional spaces by way of their experiences of the three spatial dimensions they inhabit. For example, the vertices of the two-dimensional geometric object known as the Penrose tiling are sometimes understood as projections of a five-dimensional cubic lattice in a similar way to how three-dimensional objects project two-dimensional shadows.³ These examples are not isolated instances; many more mathematical metaphors reveal themselves once one starts looking out for them.

Another questionable aspect of the traditional view is its suspicion of and hostility towards metaphor. Historically, several authors have spoken out against this traditional dismissiveness, claiming that metaphor has been significantly undervalued and mischaracterized. These scholars argue that metaphor should be embraced as a fundamental and pervasive part of human experience, not seen merely as an obfuscating derivative of literal language that should be avoided whenever possible. The advances and increased interest in language scholarship that occurred over the past century have correspondingly generated a substantial literature on metaphor, much of which views metaphor in a more positive light than earlier works.

³N.G. de Bruijn, “Algebraic theory of Penrose’s non-periodic tilings of the plane I,” *Indagationes Mathematicae* 84.1 (1981): 40.

Some contemporary authors even claim that metaphor is conceptual in nature and therefore frequently *precedes* literal language rather than being derived from it. If metaphor is a basic cognitive mechanism that helps structure our conceptual system then it is plausible that metaphor could play a constitutive role in our understanding of mathematics. Even if one rejects the idea that metaphor is conceptual, it seems that metaphor is a more complex, legitimate, and widespread phenomenon than was previously suspected.

While scholarship of the last century generally improved metaphor's reputation, it simultaneously brought mathematics down to earth a little. Mathematics was revered since the time of the Ancient Greeks as the closest we flawed, mortal humans could come to knowing objective Truth. The dramatic mathematical advances of the nineteenth century revealed that mathematics is more varied, expansive, and complicated than was previously thought. In particular, the discovery of non-Euclidean geometries called into question the long-held belief that mathematical axioms are uniquely self-evident. These developments, among others, brought about a foundational crisis in mathematics at the end of the 1800s that prompted philosophers and mathematicians to search for a way to ground and unify an increasingly abstract and voluminous discipline. While a variety of popular foundational theories have been defended, all of them are controversial in some way and there is no consensus; thus, over a century later, the foundational crisis still lacks definitive resolution. What seems clear is that mathematical results such as the non-Euclidean geometries and Gödel's incompleteness theorems have shown that mathematics is less certain and absolute than was once believed.

If mathematics and metaphor are not antipodal then the question remains of how they are related to each other. This dissertation argues that important connexions exist between mathematics and metaphor, and that exploring and developing these connexions improves our understanding of both topics. On the one hand, metaphor and analogy seem to comprise a fundamental mode of human reasoning. This is particularly evident when one considers that we frequently learn by understanding the unknown in terms of the known. Insofar as metaphorical reasoning is basic and ubiquitous, one expects it would play some role in mathematics. Conversely, the modeling capabilities of mathematics are justifiably renowned; it thus seems reasonable that one could, to at least some extent, mathematically model metaphor. Both of these approaches are considered below, though more emphasis is placed

on the former for several reasons. First, as mentioned above, the original motivation for this project was an observation that metaphor and analogy are frequently used in mathematics but the contribution they make generally goes unrecognized. Second, the idea that metaphor could play an essential role in mathematics remains controversial and unpopular, whereas significant — and increasing — amounts of effort and resources are allocated to developing mathematical models of communication and language. There is thus more need for people to defend and develop the former than the latter. Third, developing a robust mathematical model of metaphor seems too massive an undertaking to constitute a suitable doctoral project; in any case, neither my research interests nor my expertise readily lend themselves to such modeling.

This research project is fundamentally interdisciplinary. While the questions being asked are primarily philosophical in nature, the project also has a clear mathematical aspect. Additionally, perhaps because metaphors are most apparent and most applicable in mathematical development, mathematics educators constitute a significant proportion of those sympathetic to this project. Certainly, the most immediate applications of my research seem to be educational. The fifth chapter of the dissertation employs a somewhat radical interdisciplinary methodology insofar as mathematical concepts are utilized in non-mathematical ways. Even if we set aside the mathematical aspects of the project, the subgoal of understanding metaphor is itself an interdisciplinary undertaking. To various extents, the dissertation draws upon resources and evidence from cognitive science, linguistics, semiotics, biology, anthropology, and other related disciplines; however, it should be noted that this document is not intended as a work in any of these fields. A brief overview of the dissertation will help bring this all together.

Chapter 2 provides an historical overview of the traditional understanding of metaphor. The ancient history of the word “metaphor” is traced from its pre-Socratic origins through to Aristotle, who provides the first explicit account of metaphor as a figure of speech. Aristotle’s writings form the basis of the traditional view of metaphor developed by science-obsessed early modern and modern philosophers who took it to be an unnecessary linguistic distraction with no place in science or philosophy. While most scholars of the time endorsed this traditional viewpoint and its emphasis on literal truth, a small minority of poets and philoso-

phers disputed it, arguing instead that metaphor is an essential device for creative expression and allows for the expansion and development of language. The twentieth century brought increased interest in explaining metaphor and, consequently, a variety of novel philosophical theories that remain popular today. Though these theories continue in the literal-truth tradition, they also tend to be more sympathetic to the creative minority. Ultimately, all of the theories discussed in this chapter are found to be flawed. The purpose of this synoptical chapter is to provide motivation and context for the rest of the dissertation by summarizing the most popular philosophical theories of metaphor and examining their strengths and weaknesses.

Chapter 3 examines the idea that metaphor may be fundamentally conceptual rather than merely linguistic. The deficiencies observed in chapter 2 have led to increased dissatisfaction with strictly linguistic theories and, consequently, generated support for the conceptual approach to metaphor. The core of this chapter is an examination of what is arguably the foremost theory of metaphor-as-conceptual. The conceptual metaphor theory developed by linguist George Lakoff and his cadre of collaborators — notably Mark Johnson, Mark Turner, and Rafael Núñez — is the result of an extensive research project spanning three decades and myriad publications. While the Lakovian theory incorporates a significant amount of empirical research and overcomes many of the problems with linguistic approaches to metaphor, it nevertheless remains controversial. Because every theory of conceptual metaphor must rely on an understanding of what a concept is, I begin by introducing Jesse Prinz’s seven desiderata for theories of concepts, thereby providing evaluative structure to the chapter. A few popular theories of concepts are subjected to Prinz’s criteria; exposing their weaknesses provides further motivation for the alternative Lakovian approach. A relatively detailed synopsis of Lakoff’s conceptual metaphor theory follows, distilled from a more extensive selection of publications than most other summaries of his work. An assortment of published criticisms of Lakoff are addressed; while demonstrating that conceptual metaphor theory satisfies the majority of Prinz’s desiderata constitutes an adequate defense against most of these criticisms, a few are more telling and suggest areas where the theory could use improvement. The chapter concludes by briefly noting some promising directions for development, notably incorporating the work of Cornelia Müller. While this chapter does focus nearly exclusively

on Lakovian conceptual metaphor theory, it is not strongly committed to that particular theory but only to the core idea that many metaphors are conceptual in nature.

With the necessary understanding of metaphor finally established, chapter 4 argues that metaphor plays an important role in mathematics. Motivation for pursuing alternative, metaphor-dependent theories of mathematics is derived from considering the serious flaws of the traditional camps in the philosophy of mathematics; of particular note is a pervasive, mutual inability to accommodate the strengths of rival positions. The bulk of chapter 4 is devoted to two leading theories of mathematics-as-metaphor. Lakoff and Núñez’s embodied mathematics is an offshoot of Lakovian conceptual metaphor theory, an ambitious attempt to map the constitutive conceptual metaphors underlying mathematical development and practice. Embodied mathematics has been widely criticized by mathematicians and philosophers alike; however, many of these criticisms fail insofar as they are rooted in equivocation because they depend upon a traditional linguistic understanding of metaphor. Stephen Yablo’s mathematical figuralism takes a different approach, integrating metaphor with a variety of mathematical fictionalism. Imre Lakatos’s mathematical quasi-empiricism and Brian Rotman’s semiotic approach are also briefly discussed. The chapter concludes with the suggestion that a theory of mathematics grounded in an understanding of metaphor-as-conceptual will have the significant advantage of being able to integrate aspects from both traditional and contemporary competing theories.

Chapter 5 turns the tables by considering the extent to which metaphor can be understood mathematically. Scholars have been attempting to rigorously model language and concepts since at least the time of Leibniz, and the advent of the internet has intensified interest in such research. However, this chapter does not aim to make a direct contribution to this project, nor even to provide an overview of the progress that has been made. Without intending to devalue such rigorous modeling projects, I argue in favour of the non-rigorous (that is, metaphorical) application of mathematical concepts in understanding metaphor itself. I conclude chapter 5 by presenting some novel creative mathematical metaphors that I have found useful in my quest to better understand metaphor. This approach makes chapter 5 the most interdisciplinary chapter of the dissertation. Finally, a brief concluding chapter indicates some directions for future research.

CHAPTER 2

A PHILOSOPHICAL HISTORY OF METAPHOR

It is a mistake, then, to think of linguistic usage as literalistic in its main body and metaphorical in its trimming. Metaphor, or something like it, governs both the growth of language and our acquisition of it.¹

— W.V.O. Quine

The purpose of this chapter is to provide a suitable context for the discussion of metaphor in subsequent chapters by providing some preliminary answers to the question “what is metaphor?”. Every theory of metaphor must situate itself with respect to traditional and popular understandings of metaphor by either accounting for them or else explaining why it need not. Ideally, this chapter would contain a discussion of every way metaphor has been used or spoken about, both contemporaneously and throughout its long history, in every human language and culture. For obvious reasons this is not possible; I must be selective and present a summary of representative fragments of this rich history. One limitation of scope I can admit explicitly and immediately: I will focus exclusively on metaphor in the Western tradition.² While most of the history and theories I present in this chapter have been given thorough coverage elsewhere in the philosophical and linguistic literature, I believe it is a worthwhile endeavour to distill the various sources into a single text. Moreover, though similar compilations exist, none have approached the question of metaphor specifically with mathematics in mind; this novel perspective will illuminate certain features of the tradition that are often disregarded, or at least unemphasized.

¹W.V.O. Quine, “A Postscript on Metaphor,” *On Metaphor*, Ed. Sheldon Sacks (Chicago: U Chicago P, 1979), 160.

²It could be extremely rewarding to research the origin of the concept of metaphor in other languages and cultures distant from our own tradition. In particular, it would be worth investigating whether the concept of metaphor arose independently in other cultures prior to their contact with the Greeks. The results of such research could have a significant impact on our understanding of metaphor. Such an undertaking would deviate too far from my current areas of expertise and is thus beyond the scope of this dissertation.

It would be useful to have a common preliminary working definition of metaphor to use as an origin. It need not be perfect or final, it need only correspond to a core set of rudimentary thoughts about what metaphor is. I recall first learning about metaphor in elementary school language arts lessons, having been taught a definition something like the following: “A metaphor is a comparison between two things that does not use the words ‘like’ or ‘as’.” This common schoolbook definition can be seen as a simplified version of one found in the *Oxford English Dictionary (OED)*:

metaphor, n.

1. A figure of speech in which a name or descriptive word or phrase is transferred to an object or action different from, but analogous to, that to which it is literally applicable; an instance of this, a metaphorical expression. Cf. METONYMY n., SIMILE n.
2. Something regarded as representative or suggestive of something else, esp. as a material emblem of an abstract quality, condition, notion, etc.; a symbol, a token. Freq. with for, of.³

This definition will suffice as a starting point. It manages, however, to also introduce a difficulty that will plague the investigation: the word “metaphor” has several related but inconsistent senses. In the *OED* definition above, the first sense limits “metaphor” to certain instances of figurative language whereas the second sense is much broader, encompassing all language insofar as it is representational, and certainly extending to artistic works, etc.; the second sense can account for metaphors of the first type but the converse relation does not hold.

Each of the above senses seem to capture some of what is meant by “metaphor” without being exhaustive; the question thus arises how to best develop and refine a philosophical understanding of metaphor. Considering a few explicit examples would not do much to further our understanding at this point; such a process could not resolve the above difficulty, as good examples for both senses can be found.⁴ This is not to say that examples are of no use, or that this chapter will be devoid of them, only that examples alone cannot accomplish everything desired. Another way I could refine our understanding of metaphor is by following up on the associated terms suggested by the *OED*: considering concepts close to METAPHOR

³“metaphor, n.” *OED Online*, Jun. 2008, Oxford UP, 2 Feb. 2009.

⁴Indeed, the *OED* is constructed based on word usage evidence, and representative quotations are provided as part of each entry to demonstrate and justify each listed sense of the word.

can bring some of its boundary into sharper focus.⁵ This approach is one of the avenues I will pursue in chapter 3. However, I think that the best way to begin is to explore the etymology and history of the word “metaphor.” Oftentimes, knowing the origin of a word will help in understanding its modern use, though it would be a mistake to assume that etymology is or must be the arbiter of contemporary meaning.

2.1 Ancient Greece

The word “metaphor” derives from the ancient Greek word *μεταφορά*. This word combines the prefix *μετα* with the word *φορά* to mean “a carrying across” or, less strictly, “a transfer.”⁶ The noun *μεταφορά* is a cognate of the verb *μεταφέρω*, a word that occurs more frequently in the ancient Greek texts.⁷ Though he does not use the word himself, both the prefix and root verb of *μεταφέρω* are found already in Homer, suggesting that they could have been conjoined in speech centuries before the eldest surviving written instances.⁸ The verb *μεταφέρω* remains in use in modern Greek; it can be found, for instance, on the side of moving vans that can be hired to *transfer* one’s possessions *across* town.⁹ Surprisingly, while the ancients used this verb to speak of the transfer of duties between individuals and the transfer of funds in accounts, I could not find a record of any of them using this word to speak of the transfer of physical objects.¹⁰ The use of *μεταφορά* that is of primary interest here concerns linguistic transfers; despite the preceding comment, one might observe that the conception of linguistic metaphor could itself be seen as metaphoric insofar as it does not involve any physical carrying.

There is one other noteworthy sense of the word *μεταφορά*. It is one of the few surviving uses of the noun form from ancient Greece. The lengthiest known fragment of an obscure

⁵The convention of referring to concepts, conceptual domains, conceptual metaphors, and other cognitive mechanisms using small capital letters is standard in the literature on conceptual metaphor, and is adopted throughout this document.

⁶“metaphor, n.” *OED Online*. It is worth noting that the English word “transfer” comes from the Latin *transfere*, which is a direct translation of *μεταφορά* (“transfer, v.” *OED Online*).

⁷George Henry Liddell and Robert Scott, *A Greek-English Lexicon*, 9th ed. (Oxford: Clarendon, 1940).

⁸Liddell and Scott.

⁹Guy Deutscher, *The Unfolding of Language* (New York: Metropolitan, 2005), 117.

¹⁰Liddell and Scott.

poet of the third century BCE named Nicomachus portrays a cook speaking to his employer on the subtleties of his art. The cook tells the man that a “fully trained” cook must have mastered a large number of arts before studying cooking, including astronomy, medicine, and — significantly — geometry. When describing the application of each of these arts in cooking, the cook uses the term *μεταφορά*: the skills are “carried over” into cooking. That is, when astronomy is used to predict the tides in aid of selecting prime seafood or geometry is used to optimize the layout of the kitchen, the application of the skill within cooking is a metaphor insofar as it is a transference.¹¹ This is interesting because it constitutes another abstract variety of transference, a transfer of skills rather than of physical objects or of words or meanings; it suggests that, in some sense, the application of mathematical skills might be seen as involving a kind of metaphor, a view that provides motivation for the position defended in chapter 4. I now finally turn my attention to linguistic metaphor in ancient Greece.

Instances of linguistic metaphor are found in the most ancient Greek writings; that is, in Homer. Both the *Iliad* and the *Odyssey* offer up many examples of simile and metaphor.¹² Myriad examples of metaphor abound in ancient poetry, and one need not look further than the writings of Heraclitus to see they figure in the writings of philosophers as well.¹³ The linguistic phenomenon known as *μεταφορά* was clearly available to be named and conceptualized for at least several centuries before the earliest surviving use of this noun. Many historical accounts of metaphor start with Aristotle, with good reason — he was the first to give an explicit philosophical account of metaphor. I shall examine Aristotle’s writings presently. However, it is known that Aristotle was not responsible for coining the term *μεταφορά*; my investigation thus begins a few years earlier with the man responsible for the

¹¹Athenaeus, *The Learned Banqueters*, Ed. and trans. S. Douglas Olson, Vol. 3 (Cambridge: Harvard UP, 2008), 290.

¹²John Kirby, “Aristotle on Metaphor”, *American Journal of Philology* 118 (1997): 521. More on simile in the discussion of Aristotelian metaphor below, and later in chapter 3. For now, recall the schoolbook definition: A simile is a comparison between one thing and another that *does* use either “like” or “as.”

¹³For example, see Fragment 52: “Time is a child playing a game of draughts” (Freeman 28). Unfortunately, my knowledge of the ancient Greek language is limited; thus, I must depend on published translations throughout this chapter. This introduces a familiar swarm of philosophical questions concerning translation. Not only will I disregard these questions as being more or less inconsequential to the task at hand, I will occasionally draw upon multiple translations of a single work for the sake of clarity, thereby strengthening my arguments.

earliest extant usage of the noun *μεταφορά*: Isocrates.¹⁴

Isocrates (436–338 BCE) was an Athenian rhetorician of some repute; his school was the main competition for Plato’s Academy.¹⁵ His *Evagoras* is an eulogistic oration addressed to King Nicocles of Salamis, son of Evagoras; based on this fact, it is assumed to have been written circa 374 BCE, the year of Evagoras’ death.¹⁶ Near the beginning of this eulogy, Isocrates explains why eulogies are typically written in verse rather than in the prose he intends to adopt:

To the poets are granted numerous ornaments [of language], for...they can express themselves not only in ordinary language, but also by the use of foreign words (*xenais*), neologisms (*kainois*), and metaphors (*metaphorais*)...but to writers of prose none of such [resources] are permitted: they must strictly use both words and ideas [of a certain category:] (1) of words, only those that are in the [ordinary] language of the *polis*; (2) of ideas, only those that are closely relevant to the matter at hand.¹⁷

Several important points can be gleaned from this passage. According to Isocrates, there are two kinds of language: everyday, discursive language and ornamented, poetical language. Metaphor is one of the kinds of linguistic ornament that distinguishes poetical language from everyday language. Thus, language is deviant insofar as it contains metaphors and the use of metaphor is forbidden to everyone except poets. Unfortunately, this passage does not explain what Isocrates meant by “metaphor,” though it does say something about what it is *not*; it is not until Aristotle that a definition of metaphor is provided. John Kirby suggests that Isocrates may have had a very different notion of metaphor from Aristotle, one that only refers to the extended poetical similes found in Homer: “it would explain why he restricts *metaphora* to verse while at the same time using, in his own prose, what modern thinkers would call metaphor.”¹⁸ Whether Kirby’s suggestion is correct or not, it is easy to see that Isocrates is dismissive of “metaphor,” whatever the term means to him. Though Isocrates said relatively little about metaphor and is rarely mentioned in modern philosophy, his views seem to have had an influence on the tradition, as we shall see. There is one other

¹⁴It is plausible that Isocrates’ legendary rhetorician forebears Corax and Tisias may have written about metaphor; however, none of their works survive (Hinks 62).

¹⁵Isocrates, *Cyprian Orations*, Ed. Edward Forster (Oxford: Clarendon, 1912), 9–11.

¹⁶Isocrates 21.

¹⁷Translated in Kirby 523–4. Interspersed parenthetical Greek words removed to enhance readability.

¹⁸Kirby 526. For one example of an extended Homeric simile see *Iliad* 4.274–82.

pre-Aristotelian philosopher that warrants attention due to his profound influence on later philosophers of metaphor.

Plato (429–347 BCE) did not even use the word *μεταφορά* in his writing, let alone provide us with a theory of the phenomenon. However, a few select passages from his dialogues suggest what some of his thoughts on the matter might have been. These passages are certainly worth considering if for no other reason than their significant influence over subsequent philosophers. It is relevant to my particular interests that Plato’s theories make a significant contribution to the development of the traditional academic spectrum, where mathematics and the imitative arts are located on opposite ends. This antithetical relationship is best illustrated in the *Republic* and is grounded in the theory of Forms. Plato’s theory of Forms holds that everything from physical objects to spatio-temporal instantiations of abstract notions like “justice” are approximations of perfect Forms that transcend the physical realm.¹⁹ In Book X of the *Republic*, Socrates argues that because a painting of a bed is not a bed, and a bed is not the Form of bed, the works of poets and other imitative artists are three removes from the truth.²⁰ Because “all poetical imitations are ruinous to the understanding of the hearers,”²¹ Plato’s Socrates would either drastically censor all poets or exile them from his Republic: “we must remain firm in our conviction that hymns to the gods and praises of famous men are the only poetry which ought to be admitted into our State”²² On the other hand, Plato reveres mathematics as one of the best kinds of human knowledge, the most accurate concrete approximation of the Forms that the human mind can know: “the knowledge at which geometry aims is knowledge of the eternal, and not of aught perishing and transient... geometry will draw the soul towards truth, and create the spirit of philosophy.”²³ Elsewhere, Plato directly situates mathematics in opposition to poetry, claiming that “measuring and numbering and weighing” help fend off the bewitching falsehoods of the imitative arts.²⁴ Combining the above results with Plato’s unwavering pursuit of the truth

¹⁹There is certainly more to this theory than can be summed up in a single sentence, but this gloss shall suffice for my purposes.

²⁰Plato, *Republic*, Trans. Benjamin Jowett, *The Internet Classics Archive*, 597e.

²¹Plato, *Republic* 595b.

²²*Republic* 607a.

²³*Republic* 527b.

²⁴*Republic* 602d.

and Isocrates' idea that metaphor is one of the characteristic devices of poetry generates a few important implications. First, the resultant diametric opposition between poetry and mathematics suggests one explanation why relatively few scholars have looked for connexions between the two. Secondly, and more importantly, if metaphor is exclusive to poetry and poetry is far removed from the truth, then one might deduce that Plato held metaphor to be generally disreputable and worthy of suspicion; certainly, many later thinkers incorporate such a suspicion of metaphor into their own philosophy, as we shall see. However, there are good reasons to believe this does not accurately reflect Plato's position.

The second inference cannot hold because it seems quite clear that Plato did not accept the Isocratic idea that metaphor is exclusive to poetry; after all, Plato's own writing is rife with metaphor. A particularly relevant passage can be found in his *Phaedrus*:

To tell what [the soul] really is would be a matter for an utterly superhuman and long discourse, but it is within human power to describe it briefly in a figure; let us therefore speak in that way. We will liken the soul to the composite nature of a pair of winged horses and a charioteer. Now the horses and charioteers of the gods are all good and of good descent, but those of other races are mixed; and first the charioteer of the human soul drives a pair, and secondly one of the horses is noble and of noble breed, but the other quite the opposite in breed and character. Therefore in our case the driving is necessarily difficult and troublesome.²⁵

This excerpt shows, not only did Plato use metaphor, he also held that some truths were *best* expressed using metaphor.²⁶ Indeed, Plato was denounced by several ancient critics specifically for his use of metaphor.²⁷ Does juxtaposing this revelation with the passages from the *Republic* expose Plato as a hypocrite? Not at all. Plato clearly realizes that metaphor is not exclusive to poetry; it is also used in philosophy at the very least. His attitude toward metaphor is better understood as respectful caution than outright suspicion: he recognizes metaphor as a powerful tool which, on the one hand, can be used to convey the most important and difficult truths but, on the other hand, can obfuscate even the simplest and most basic facts.

²⁵Plato, *Phaedrus*, Trans. Harold N. Fowler, *Plato in Twelve Volumes* (Cambridge: Harvard UP, 1925), 246a–b.

²⁶Some readers might object that the above passage would be better classified as simile; I believe this distinction is inconsequential to the point I am making. Indeed, even if one accepts Kirby's suggestion regarding Isocratic metaphor — that is, that the term only applies to extended similes such as those found in Homer — this passage retains its evidential strength.

²⁷Kirby 530.

Thus far, inferences about Plato's view of metaphor have been made based on his discussion of mimetic poetry as well as on observations of his own abundant use of metaphor. One other feature of his writing deserves mention. As has already been noted, Plato never used the noun *μεταφορά* in his dialogues. He did, however, use the verb *μεταφέρω* in several places, though never to refer to a transfer of words or meanings.²⁸ His most interesting use of the word occurs in the *Timaeus*. In the early pages of the dialogue, Socrates gives a condensed summary of his ideal State, and then reminds Timaeus, Hermocrates, and Critias that he had set them the task of depicting this ideal city at war.²⁹ In response to this challenge, Critias famously relates the story of Atlantis: the legendary kingdom once attacked the Hellenic people, but the Atlanteans were defeated by the Athenians and lost their island to the ocean depths. Critias then makes the suggestion that the three satisfy Socrates' request in the following way: "The city and citizens, which you yesterday described to us in fiction, we will now transfer (*μετενεγκόντες*) to the world of reality. It shall be the ancient city of Athens, and we will suppose that the citizens whom you imagined, were our veritable ancestors."³⁰ The transfer here is of an abstract or fictional entity into an actual scenario.³¹ This use of the verb *μεταφέρω* seems similar to Nicomachus' use of *μεταφορά*, and both are relevant to my project, as shall become apparent in chapter 4.³²

Aristotle (384-322 BCE) is the first scholar to give an explicit account of metaphor. The bulk of his account is split between the *Poetics* and Book III of the *Rhetoric*. In the *Poetics*, Aristotle gives the following definition: "Metaphor consists in giving the thing a name that belongs to something else; the transference being either from genus to species, or from species to genus, or from species to species, or on grounds of analogy."³³ One important point arises

²⁸This is not strictly true; in his *Critias*, Plato uses the word *μεταφέρω* to refer to translation between languages (Kirby 528). This usage foreshadows some ideas that will come up in chapter 4, but at present seems distantly removed enough from the schoolbook definition to be merely noted.

²⁹Interestingly, Socrates describes his desire to hear their reply by metaphorically comparing it to the desire one might have to breathe animating life into a static painting of animals.

³⁰Plato, *Timaeus*, Trans. Benjamin Jowett, *The Internet Classics Archive*, 26c-d.

³¹The four conversation partners take the Atlantis story as *fact* (Plato, *Timaeus* 20d).

³²Anticipating the discussion of conceptual metaphor in chapter 3, Nicomachus' and Plato's underlying conceptual metaphors seem to be COOKING IS GEOMETRY and ANCIENT ATHENS IS THE SOCRATIC IDEAL CITY respectively.

³³Aristotle, *Poetics*, Trans. I. Bywater, *The Complete Works of Aristotle*, Ed. Jonathan Barnes (Princeton: Princeton UP, 1984), 1457b6-7. The word "epiphora" is used in this definition, and this makes it close to tautological. Fortunately, one can flesh out this definition by considering other passages. Moreover, following

immediately from this definition: for Aristotle, metaphor is primarily a transfer of names or words, not of meanings or aught else. He follows this definition with illustrative examples of each of these four types. It has been noted that, in modern terminology, most examples of the first three kinds would be classified as cases of metonymy or synecdoche rather than metaphor.³⁴ Moreover, it seems that Aristotle's second example ("Truly has Odysseus done ten thousand deeds of worth") could also be seen as a case of hyperbole.³⁵ It is clear that the connexions between metaphor, metonymy, and synecdoche must be considered in greater detail; I will postpone this task to chapter 3. Instances of the fourth type of metaphor, those based in proportional analogy ("As old age is to life so is evening to day") seem to be more typically metaphorical according to the schoolbook definition.³⁶ These analogical metaphors have several noteworthy features.

Aristotle explains that his fourth type of metaphor is symmetrical: "the proportional metaphor must always apply reciprocally to either of its co-ordinate terms."³⁷ For example, one may either describe evening as the "old age of the day" or describe old age as "the sunset of life."³⁸ However, Aristotle also says that proportional metaphors can be used to name the unnamed; indeed, this is one of the most significant properties of metaphor. For example, when a new kind of beetle with pointy protuberances coming out of its head is discovered, the entomologist may note "as antlers are to a stag so *Xs* are to this beetle" and metaphorically refer to the protuberances as the antlers of the beetle. This variety of proportional metaphor where one of the terms is unnamed seems to be fundamentally asymmetrical in that no name exists to be transferred reciprocally. Considerations of symmetry similar to these will play a role in any viable theory of metaphor.

Bickerton, Leezenberg notes that Bywater's translation is erroneous: the phrases "the thing" and "belongs to something" have no analogues in the original Greek. These additions arguably illegitimately attribute several presuppositions regarding language to Aristotle that have skewed the interpretations of scholars over the last century (Leezenberg 36n).

³⁴Christof Rapp, "Aristotle's Rhetoric", *Stanford Encyclopedia of Philosophy*.

³⁵*Poetics* 1457b11–12; qtd. in Kirby 533. Elsewhere, Aristotle notes "Successful hyperboles are also metaphors." (*Rhetoric*, Trans. Roberts 1413a20). His argument for this claim is somewhat unclear, perhaps he is simply agreeing with my observation.

³⁶Aristotle, *Poetics* 1457b22–23. Aristotle notes elsewhere that "Of the four kinds of metaphor, the most popular are those based on proportion" (*Rhetoric*, Trans. Freese 1411a1).

³⁷*Rhetoric* 1407a14–16.

³⁸Aristotle, *Poetics* 1457b16–24. Note also that the first and second types of metaphor are reciprocal to each other and that the third is self-reciprocal.

The ancient Greek for simile is *εἰκὼν*, a word that was also used to refer to visual resemblances and is thus the ancestor of the English word “icon.”³⁹ Aristotle explicitly explains how metaphor (*μεταφορά*) and simile (*εἰκὼν*) are connected: “the simile... is a metaphor, differing from it only in the way it is put; and just because it is longer it is less attractive.”⁴⁰ Aristotle continues his discussion of simile a few pages later, noting that “Successful similes... always involve two relations like the proportional metaphor... a simile succeeds best when it is a converted metaphor.”⁴¹ Thus, the connexion between simile and proportional metaphor is made explicit. It is noteworthy that Aristotle considers simile to be a subspecies of metaphor; Cicero, Quintilian, and much of the ensuing tradition hold precisely the opposite view, that all metaphors are elliptical similes.⁴² Note that Aristotle’s claim that all similes are metaphors does not logically entail that all metaphors are similes. The *OED* definition of simile varies very little from the ancient understanding: “A comparison of one thing with another, esp. as an ornament in poetry or rhetoric.”⁴³ I will return to the relationship between simile and metaphor in chapter 3.

Aristotle differs from those who follow in his tradition by holding simile to be a species of metaphor. Another divergence from this tradition also constitutes a divergence from his predecessor Isocrates. Whereas Isocrates holds that metaphor is limited to poetry, Aristotle claims the following: “these two classes of terms, the proper or regular and the metaphorical — these and no others — are used by everybody in conversation.”⁴⁴ While Aristotle does note that some figures are exclusive to poetry, it is clear that metaphor is not among them: it is used in prose and in everyday speech.⁴⁵ Not only does Aristotle see metaphor as more widespread than Isocrates did, he says nothing to suggest he would have it any other way; in fact, he enthusiastically remarks that “the greatest thing by far [for a poet] is to be a master of metaphor. It is the one thing that cannot be learnt from others; and it is also

³⁹Plato used the word *εἰκὼν* with both of these meanings at various points in his corpus, but these passages are not very relevant to my purposes; I omit them for the sake of brevity. For more details, see Marsh McCall, *Ancient Rhetorical Theories of Simile and Comparison*, Chapter 1.

⁴⁰*Rhetoric*, Trans. Roberts 1410b16–18.

⁴¹*Rhetoric*, Trans. Roberts 1412b3–4.

⁴²Mark Johnson, “Introduction,” *Philosophical Perspectives on Metaphor*, Ed. Mark Johnson (Minneapolis: U of Minnesota P, 1981), 8.

⁴³“simile, n.,” *The Oxford English Dictionary*, 2nd ed., 1989, *OED Online*, Oxford UP, 18 Feb. 2009.

⁴⁴*Rhetoric*, Trans. Roberts 1404b34–36.

⁴⁵*Poetics*, 1459a10–14.

a sign of genius, since a good metaphor implies an intuitive perception of the similarity in dissimilars.”⁴⁶ This quote shows that metaphor is not just a matter of technique, but involves a type of creativity or insight that cannot be taught. It also reinforces the idea that, for Aristotle, the metaphoric transfer of words is based upon similarity; this is an assumption that some authors have recently started to question.⁴⁷ The two quotes in this section prompt an important question: if metaphor is so widespread and important, what purposes does it serve?

I have already discussed one important role metaphor plays: the metaphorical naming of unnamed phenomena and entities, a process sometimes called catachresis. In discussing the further uses of metaphor, it is important to note the following: “It should be observed that each kind of rhetoric has its own appropriate style. The style of written prose is not that of spoken oratory, nor are those of political and forensic speaking the same.”⁴⁸ Thus, for each different mode of linguistic communication, metaphor may play a slightly different role in the corresponding rhetorical style. Aristotle provides the following general account:

The excellence of diction is for it to be at once clear ($\sigma\alpha\phi\eta$) and not mean ($\mu\eta\tau\alpha\pi\epsilon\iota\nu\eta$).⁴⁹ The clearest indeed is that made up of the ordinary words for things, but it is mean. . . On the other hand the diction becomes distinguished and non-prosaic by the use of unfamiliar terms, i.e. strange words, metaphors, lengthened forms, and everything that deviates from the ordinary modes of speech. But a whole statement in such terms will be . . . a riddle if made up of metaphors. . . A certain admixture, accordingly, of unfamiliar terms is necessary.⁵⁰

Thus, a second general purpose of metaphor is to accentuate truths by making them more distinct and appealing. The trick, as is often the case with Aristotle, is to aim for the perfect balance between the extremes; in this case, to seek out the perfect combination of ordinary words and figures of speech that constitutes an equilibrium between unsophisticated clarity and unclear sophistication.

⁴⁶Aristotle, *Poetics* 1459a5–8. According to the *OED*, the word “genius” comes from the Greek $\gamma\acute{\iota}\gamma\nu\epsilon\sigma\theta\alpha\iota$ by way of Latin and has connotations of creation. It is used here as a translation of the Greek word $\epsilon\nu\phi\nu\lambda\acute{\alpha}\varsigma$ which, according to the Liddell-Scott Greek-English Lexicon, means natural goodness of shape, parts, mind, or disposition (Liddell and Scott).

⁴⁷See, for example, Johnson, “Introduction” 6. However, it seems to me that the phrase “similarity in dissimilars” leaves some room for interpretation. I will return to this in chapter 3.

⁴⁸Aristotle, *Rhetoric*, Trans. Roberts 1413a3–5.

⁴⁹The word “mean” has enough meanings (!) in English to cause some confusion here. To help clarify, note that other sources translate this word as “commonplace” or “low” or “pedestrian” (Liddell and Scott).

⁵⁰*Poetics*, Trans. Bywater 1458a18–32.

Elsewhere, Aristotle says that “Metaphor... gives style clearness, charm, and distinction as nothing else can.”⁵¹ This quote suggests, contrary to the passage in the previous paragraph, that some metaphors can simultaneously increase both distinction and clarity. One hypothesis is that metaphors may sometimes increase clarity by means of improved efficiency of communication; an apt, brief metaphor can be richly meaningful.⁵² The idea of aptness — that instances of metaphor can be better or worse — is one of the key issues in the study of metaphor. In the *Rhetoric*, Aristotle argues

the fallacious argument of the sophist Bryson [says] that there is no such thing as foul language, because in whatever words you put a given thing your meaning is the same. This is untrue. One term may describe a thing more truly than another, may be more like it, and set it more intimately before our eyes. Besides, two different words will represent a thing in two different lights; so on this ground also one term must be held fairer or fouler than another.⁵³

That Aristotle argues this point is not surprising; indeed, one of the fundamental justificatory assumptions of rhetoric is that there is more than one way to express a given truth. Metaphor is certainly one important mechanism that allows for this variation in expression. While people ought to accept or reject ideas primarily, or even solely, on the basis of truth, the particular wording one uses in communicating a truth may play an important supporting role in convincing an audience.⁵⁴ If metaphor can improve communication, one might expect that it can also hinder it. Aristotle goes on to explain that there are several common ways for metaphors to fail to be appropriate: they may be too tragic, too farfetched, too ridiculous.⁵⁵ This brief glimpse into Aristotle’s views on what makes for an apt metaphor provides a starting point for a contemporary understanding.

This discussion of the second purpose of metaphor originated with a crucial distinction. Recall that Aristotle presents an opposition between unfamiliar terms and ordinary words; this distinction is identical to the one made by Isocrates, though Aristotle is friendlier to

⁵¹ *Rhetoric*, Trans. Roberts 1405a8–9.

⁵² To slightly alter a cliché to fit my purposes, a figure [of speech] is worth a thousand words.

⁵³ *Rhetoric*, Trans. Roberts 1405b11–15.

⁵⁴ For example, by using language that is appealing to a specific audience, a speaker may attract their undivided attention and make them more receptive to the truth being communicated through humour, pity, fear, etc. Note that though these techniques of language have important positive uses, they may also be used fallaciously to convince an audience of falsehoods — see, for example, the fallacy of mob appeal.

⁵⁵ *Rhetoric*, Trans. Roberts 1406b6.

unfamiliar words. While Aristotle has much to say on metaphor in the *Rhetoric* and the *Poetics*, he says almost nothing about ordinary words. In the *Poetics*, he says “By the ordinary word, I mean that in general use in a country.”⁵⁶ However, this does not explain very much as it says nothing explicit about the ultimate grounding of this general usage.⁵⁷ Aristotle’s contrast between ordinary and unfamiliar words bears a strong resemblance to the distinction between literal and figurative language utilized by scholars in theorizing about metaphor for at least the last four hundred years.⁵⁸ As “metaphor” is sometimes defined as being synonymous with “non-literal,” it is somewhat surprising to discover that relatively little explicit consideration has been given to the question “what is literality?” by those who study metaphor. The second purpose of metaphor — to contribute attractive and enlivening sophistication to language — raises several deep and important issues that require attention, particularly those related to aptness and literality.

A third purpose of metaphor is that it can be used to teach. This usage is related to catachresis in that every thing a person encounters was once new and nameless to them. That metaphor has a didactic usage is revealed by Aristotle in the following passage:

we all naturally find it agreeable to get hold of new ideas easily: words express ideas, and therefore those words are the most agreeable that enable us to get hold of new ideas. Now strange words simply puzzle us; ordinary words convey only what we know already; it is from metaphor that we can best get hold of something fresh.⁵⁹

Whenever a person encounters a new idea, they tend to compare it to things and ideas they are familiar with, looking for similarities. This process can provide the foundation for an analogical metaphor that allows one to speak and reason about the novel idea. Under this

⁵⁶Aristotle, *Poetics* 1457b3–4.

⁵⁷It is possible that Aristotle says something more about ordinary speech elsewhere, in some part of his corpus I am less familiar with. I suspect that, like many later scholars who discussed metaphor, Aristotle has at least partially taken for granted that the idea of ordinary language is obvious and needs little further definition. Whether they are privy to explication unknown to me or have made inferences from his broader theory, the editors of Liddell and Scott’s Greek-English Lexicon define Aristotle’s term for ordinary words, *κύριον*, as “the *real* or *actual*, hence *current*, *ordinary*, name of a thing” (Liddell and Scott).

⁵⁸The English word “literal” derives from the Latin word *litteralis*, an adjective meaning roughly “pertaining to [alphabetic] letters.” In English, the relevant definition for my purposes is “[pertaining] to the etymological or the relatively primary sense of a word, or to the sense expressed by the actual wording of a passage, as distinguished from any metaphorical or merely suggested meaning.” This meaning derives from an earlier usage describing a mode of Biblical interpretation that contrasts with the mystical or allegorical (“literal, adj. and n.” *OED Online*).

⁵⁹*Rhetoric*, Trans. Roberts 1410b9–13.

view, it becomes clear that one way a teacher may induce learning is by drawing analogies between ideas familiar to their students and unfamiliar material from the curriculum. Because a person may fail to be taught to craft good metaphors, those who are predisposed towards metaphor mastery have an advantage both in teaching and in learning as they have the potential to create powerful new metaphors as the need arises.⁶⁰ Elsewhere, Aristotle calls into question the value of metaphor and other stylistic devices in teaching: “in every system of instruction there is some slight necessity to pay attention to style; for it does make a difference, for the purpose of making a thing clear, to speak in this or that manner; still, the difference is not so very great, but all these things are mere outward show for pleasing the hearer.”⁶¹ The above two quotations seem difficult to reconcile under the assumption that metaphor is solely stylistic: they agree that metaphor has some place in instruction and learning, but differ drastically on the importance of that role.

The only way I can see to resolve this tension is by explaining why the second quotation does not apply to metaphor. One option is to consider the possibility that metaphor has non-stylistic uses; though Aristotle’s discussion of metaphor is dominated by considerations of its sophisticating purpose, the brief discussion of catachresis makes this hypothesis promising, and I will give it further consideration below. Another possibility is that Aristotle is using the word “style” as a metaphor of the first kind; that is, he is transferring the word for a genus, “style,” to one of its species, “delivery.” Thus, by saying that style does not play a big role in instruction, Aristotle might only mean that teachers need not have sonorous voices and vocal skills on par with actors. This interpretation seems somewhat spurious but is supported by the fact that the second quotation is taken from a passage which primarily focuses on delivery; unfortunately, my knowledge of Greek is not sufficient to further test this hypothesis. This incongruity seems of little consequence given that most of Aristotle’s writing on metaphor supports the first quotation, and the second quote does not even contain the word “metaphor”; why then am I giving it thorough consideration? The reason is that the sentence immediately following the second quotation contains one of the only references to mathematics in the *Rhetoric*: “Nobody uses fine language when teaching geometry.”⁶²

⁶⁰ Aristotle, *Poetics* 1459a5–8.

⁶¹ *Rhetoric*, Trans. Freese 1404a9–13.

⁶² Aristotle, *Rhetoric*, Trans. Roberts 1404a13. The only other reference to mathematics in the *Rhetoric*

Insofar as mathematics can be considered a mode of communication, it too must have its own associated rhetorical style. This quote suggests that Aristotle may have held that geometry should be conducted entirely in ordinary language, that metaphors play no role in geometrical education.⁶³ One might question whether Aristotle can be considered an expert regarding mathematics instruction; while he taught a great number of disciplines, none of his surviving works are specifically about mathematics. As one of the motivating intuitions of my project is that mathematical practice necessarily involves metaphor, the apparent incongruity in Aristotle's writing on the use of metaphor in teaching discussed in the last two paragraphs is of great interest.

There are several reasons why I have given a relatively lengthy treatment to Aristotle's theory of metaphor. First, it is justifiably considered to be the original account of metaphor; this fact alone makes it interesting and worthy of discussion. Furthermore, the above summary of his account shows it to be both descriptive — as it provides a definition for metaphor and situates the concept with respect to related concepts — and prescriptive — insofar as it explains some of the appropriate and inappropriate uses of metaphor. This comprehensive scope is rare among theories of metaphor and provides further incentive to situate contemporary theories in relation to Aristotle. In particular, Aristotle motivates discussion of the symmetry features of metaphor, of the relation of metaphor to simile and analogy, of what makes an apt metaphor, of what constitutes literality and how it is related to metaphor, and of the use of metaphor in didactics.

Many academics have chosen to build their account of metaphor in a fairly narrow tradition based upon Aristotle so having a solid comprehension of his theory will assist with understanding subsequent authors. Indeed, there is so little deviation from the aforementioned tradition for thousands of years that consideration of a few representative authors will be sufficiently illustrative; as Umberto Eco notes, “of the thousands and thousands of pages written about the metaphor, few add anything of substance to the first two or three

I am aware of is of less consequence at this juncture, but I quote it here for the sake of completeness: “mathematical discourses depict no character; they have nothing to do with choice, for they represent nobody as pursuing any end” (1417a19–20).

⁶³It is worth observing that another potential avenue of resolution is to consider whether Aristotle meaningfully distinguished between teaching and learning, holding that the role of metaphor is slight in the former but great in the latter.

fundamental concepts stated by Aristotle.”⁶⁴ Although Eco might not have had the same fundamentals in mind, Mark Johnson’s suggestion that “the future of metaphor is prefigured in terms of these three basic components [of Aristotle’s theory]: (i) focus on single words that are (ii) deviations from literal language, to produce a change of meaning that is (iii) based on similarities between things”⁶⁵ certainly seems to be consistent with Eco’s claim. Johnson goes on to suggest that the following definition of metaphor unifies the dominant tradition: “A metaphor is an elliptical simile useful for stylistic, rhetorical, and didactic purposes, but which can be translated into a literal paraphrase without any loss of cognitive content.”⁶⁶ While Johnson’s first quote is accurate, there seem to be some fundamental divergences between Aristotle’s theory and the tradition described by the latter quote.

A final reason for reviewing Aristotle’s theory in detail is to suggest that his theory should not be located within the tradition he is often credited with originating. There are several reasons for thinking this. First, as noted above, Aristotle believed that simile was a species of metaphor; Cicero was among the first influential thinkers to invert this relation.⁶⁷ One does not have to be the father of categorical logic to appreciate that $\{\text{similes}\} \subset \{\text{metaphors}\}$ and $\{\text{metaphors}\} \subset \{\text{similes}\}$ have very different repercussions. Second, nowhere does Aristotle say that metaphors are eliminable; rather, his suggestion that one must seek a mean between ordinary and unfamiliar language suggests the opposite. This second divergence from the tradition seems to depend in part on the first. Similarly, the third is related to the second: many scholars in the tradition not only think that metaphor *is* eliminable, they believe it *ought* to be eliminated. This view corresponds more closely to Isocrates than to Plato or Aristotle. To be sure, both Plato and Aristotle are occasionally unfriendly to poetic language, unfamiliar terms, and metaphor in their writing insofar as they can be taken to excess and used for sophistry. However, they also both expound the virtues of the moderate use of similes, metaphors, and figurative language in various types of linguistic discourse — including the philosophical. The similarities and differences between Aristotle’s theory and those of his philosophical descendants will become more apparent after further consideration

⁶⁴Umberto Eco, *Semiotics and the Philosophy of Language* (Bloomington: Indiana UP, 1984), 88.

⁶⁵Johnson, “Introduction” 6.

⁶⁶Johnson, “Introduction” 4.

⁶⁷Johnson, “Introduction” 8.

of a few representative members of the tradition.

The history of the concept of metaphor is a tree. Its roots are Aristotle and his contemporaries. A straight trunk with almost no branches stretches upward out of these roots until the twentieth century, where the first seriously acknowledged branches diverge. By the 1970s, the crown of the tree develops in earnest, with many twigs protruding from several branches. Thus, as the investigation continues through history, there is an increasing pressure to generalize and simplify rather than consider each individual twig. This is unfortunate but necessary, as a thorough chronicling is beyond the scope of this dissertation and, indeed, probably beyond the scope of *any* single paper. The primary goal of this chapter is to provide context for discussion of non-traditional contemporary theories of metaphor; considering the major trends in metaphor theory along with a few key representative figures will be adequate for this purpose. Given the way theories develop, a more or less chronological approach remains appropriate.

2.2 Early Modern and Modern Philosophy

For many centuries, theories of metaphor mostly consisted of gradually diminishing echoes of Aristotle's discussion. As already noted, the most notable deviation from Aristotle is the inversion of the genus to species relationship of metaphor and simile, an idea supported by Cicero in his *De Oratore* and in most subsequent writings on the subject. This reversal seems strongly correlated to metaphor's diminishing status in the writings of rhetoricians and theologians throughout late antiquity and the middle ages. While the role of metaphor in philosophy was at first simply deemphasized, eventually its noted positive contribution to non-poetic modes of language was entirely forgotten, eclipsed by condemnation on grounds of being a utensil of sophistry. These sentiments were taken up by many of the science-obsessed philosophers of the early modern era.⁶⁸

Hobbes (1588–1679) is an ideal representative of this tradition: he provides an extreme stance on metaphor which, upon brief reflection, reveals him as a hypocrite. While it is the theory of the commonwealth presented in the *Leviathan* that is most often discussed,

⁶⁸Johnson, "Introduction" 9–11.

the book begins with a scientifically themed discussion of human beings and their mental capacities. In the fourth chapter “Of Speech,” Hobbes roughly defines metaphor as the use of words “in other sense than that they are ordained for” and ranks it as one of the four abuses of language on the grounds of its deceptive character.⁶⁹ Furthermore, Hobbes says that metaphor is the sixth cause of absurdity in reasoning: “though it be lawful to say, for example, in common speech *the way goes or leads hither or thither, the proverb says this or that*, whereas ways cannot go nor proverbs speak, yet in reckoning and seeking of truth such speeches are not to be admitted.”⁷⁰ For Hobbes, metaphor is the non-literal and has no place in science, philosophy, or any serious, truth-seeking enterprise. This view is far less detailed than that of Aristotle. Hobbes and Aristotle agree that metaphor is non-literal, though Aristotle counts many other species of unfamiliar language. However, they clearly disagree in their prescriptions: whereas Aristotle believes metaphor use is good in moderation, Hobbes would eliminate metaphor completely, at least within academia.

Given Hobbes’ explicit unwavering rejection of metaphor, one might expect a scarcity of non-literal language in his writing. This is obviously not the case. The very title of his magnum opus, *Leviathan*, betrays a pervasive foundational metaphor: THE STATE IS A POWERFUL HUMAN. The first paragraph of Hobbes’ Introduction introduces the metaphor MACHINES ARE LIVING THINGS in detail: “what is the *heart*, but a *spring*; and the *nerves*, but so many *strings*; and the *joints*, but so many *wheels*.”⁷¹ Perhaps most condemning, the following quotation comprises the conclusion of the fifth chapter of *Leviathan*, “Of Reason, and Science”:

To conclude, the light of human minds is perspicuous words, but by exact definitions first snuffed and purged from ambiguity; *reason* is the *pace*; increase of *science*, the *way*; and the benefit of mankind, the *end*. And, on the contrary, metaphors, and senseless and ambiguous words, are like *ignes fatui* [a fool’s fire], reasoning upon them is wandering amongst innumerable absurdities; and their end, contention and sedition, or contempt.⁷²

⁶⁹Thomas Hobbes, *Leviathan*, Ed. Edwin Curley (Indianapolis: Hackett, 1994), I.iv.4. The word “ordained” potentially has problematic religious connotations. It suggests that literality might depend upon divine decree. Hobbes’ presentation of the Babel myth in I.iv.2 suggests he may be simply using the word “ordained” to mean “proper” here. I note this merely as a possible explanation of literality, albeit one that is unacceptable for my purposes.

⁷⁰Hobbes I.v.14.

⁷¹Hobbes 1.

⁷²Hobbes I.v.20. Bracketed comment included in text. *Ignes fatui* are also known as will-o’-the-wisps, ghostly lights known for luring people into bogs.

This passage beautifully portrays metaphor as the antithesis of the goods of reason and science; however, Hobbes uses metaphor in his very disparagement of metaphor! This apparent hypocrisy provides evidence in favour of the rejection of extreme anti-metaphor stances.

This conclusion may have interesting implications for mathematics. Like Descartes, Hobbes attempts to make his project rigorous by making impositions on it analogous to those found in mathematical reasoning, and these are largely responsible for his rejection of metaphor.⁷³ In his discussion on speech, Hobbes claims that “without words there is no possibility of reckoning of numbers, much less of magnitudes.”⁷⁴ He notes that people without language might have some extremely basic arithmetic and geometric skills, but would lack the ability to abstract from particular objects and figures.⁷⁵ Thus, if one accepts that mathematics depends on words, and that attempting to eliminate all metaphor from discourse in order to introduce mathematical rigor is a fool’s errand, then one might conclude that metaphor has some role to play in mathematical reasoning. This argument involves a lot of speculation, hypotheticals, and some controversial assumptions, but it gestures towards the position that will be defended in chapter 4.

Hobbes’ view of metaphor provides an illustration of what Johnson calls “the literal-truth paradigm,” a position characterized by a belief in the primacy of literal language, with metaphor being an eliminable deviation from the literal.⁷⁶ Hobbes is not the sole advocate of this paradigm; indeed, the majority of theories of metaphor are variations on this theme. While not all those who fall under the literal-truth paradigm are as hostile to metaphor as Hobbes, many of his philosophical contemporaries had similar sentiments. George Berkeley (1685–1753), for example, simply proclaims “a philosopher should abstain from metaphor.”⁷⁷ John Locke (1632–1704) is more vicious in his condemnation, stating that all figurative language is “for nothing else but to insinuate wrong ideas, move the passions, and thereby mislead the judgment” and is therefore “wholly to be avoided,” concluding that “where truth and knowledge are concerned, [figures of speech] cannot but be thought

⁷³For more on Descartes’ desire to use the rigorous deductions of axiomatic geometry as the model for all inquiry, see his *Discourse on Method* (1637).

⁷⁴Hobbes I.iv.10.

⁷⁵Hobbes I.iv.9.

⁷⁶Johnson, “Introduction” 12.

⁷⁷From *Of Motion*, quoted in Johnson, “Introduction” 12.

a great fault, either of the language or the person that makes use of them.”⁷⁸ Sometime in the eighteenth century philosophical discussion of metaphor seems to have fallen out of vogue, possibly due to stagnation; the few theories that were published tended to simply be uninspired restatements of the literal-truth paradigm expressing a general animosity towards metaphor.⁷⁹ It would take the renewed interest in the philosophy of language at the end of the nineteenth century to bring metaphor under serious philosophical scrutiny once again.

Above, I have provided a sketch of the primary traditional view of metaphor from its roots in the ancient Greeks through nearly two-and-a-half millennia of development and dormancy. However, not everybody subscribes to the literal-truth paradigm; considering some of these alternative positions will add crucial breadth to this chapter. The non-traditional views of metaphor considered in this chapter exhibit enough commonalities to justify the application of a classificatory term. As these viewpoints are unanimous in recognizing creative imagination, and metaphor in particular, as fundamentally important, I shall say they are representative of the *creative-imagination paradigm*.⁸⁰ A few general remarks are in order. First, note that simply being friendly towards metaphor is not a sufficient condition for membership in this class. Second, it is not entirely clear when this branch diverged from the main tradition. Its origins can be seen in certain passages in Aristotle.⁸¹ However, the particular views considered below originate in the eighteenth and nineteenth centuries. I suspect that there have been proponents of the creative-imagination paradigm throughout history whose beliefs have been lost for various reasons.⁸²

⁷⁸John Locke, *Essay Concerning Human Understanding*, Ed. Peter H. Nidditch (Oxford: Clarendon, 1975), III.x.34.

⁷⁹Johnson, “Introduction” 13.

⁸⁰The words “imagination” and “paradigm” should be understood in a loose and common rather than technical way here: no philosophically substantial claims were intended by the particular choice of terminology.

⁸¹For example, Aristotle on catachrestical metaphor in the *Poetics*, 1457b24–29.

⁸²Some hypotheses:

1. Because the creative-imagination paradigm tends to lend support to theories regarding the origin and development of language that could be seen as heretical, proponents of this paradigm were often not vocal in supporting their beliefs until a point in history when it was safer to do so.
2. Anyone who did write in support of the primacy of metaphor would be more likely to utilize poetical or ostentatious metaphor in that writing, thereby making it more likely to be rejected for publication by proponents of the literal-truth paradigm.
3. There seems to be a correlation between the aforementioned decrease of publications admonishing the use of metaphor and the increase in publications supporting the creative-imagination paradigm.

Where the literal-truth paradigm has the scientifically minded philosopher as an archetypal advocate, the creative-imagination paradigm has the Romantic poet. One might expect this, as poets rely on metaphor as one of their fundamental professional tools. To some extent, this claim is based upon anecdotal evidence as most poets opt to write poetry rather than treatises on metaphor, just as most practicing mathematicians would prefer to do mathematics than contemplate its foundations.⁸³ However, a few poets have left records of their beliefs on these matters. Percy Bysshe Shelley's (1792–1822) *A Defence of Poetry* is a prime example of such a record.⁸⁴ In this paper, Shelley presents the following view of metaphor:

[The language of poets] is vitally metaphorical; that is, it marks the before unapprehended relations of things, and perpetuates their apprehension, until words, which represent them, become, through time, signs for portions and classes of thought, instead of pictures of integral thoughts; and then, if no new poets should arise to create afresh the associations which have been thus disorganised, language will be dead to all the nobler purposes of human intercourse.⁸⁵

This passage claims that, far from being an ornamental deviation from the literal, metaphor actually *precedes* the literal. It also suggests that without poets to inject life into language, it would die and wither, becoming useful only for the most base tasks. This is the first time the notion of dead language has appeared in this chapter, but it becomes a pervasive theme in later writing on metaphor. This synopsis of Shelley's view of metaphor is substantiated further down the page:

In the infancy of society every author is necessarily a poet, because language itself is poetry... Every original language near to its source is in itself the chaos of a cyclic poem: the copiousness of lexicography and the distinctions of grammar are the works of a later age, and are merely the catalogue and the form of the creations of Poetry.⁸⁶

⁸³One anecdotal source is provided by Johnson: "It was Romantic artists and poets, rather than philosophers, who preserved and celebrated the notion of creative imagination" ("Introduction" 14–5). However, Johnson provides no explicit examples to back up this claim.

⁸⁴Shelley's paper is a response to his friend Thomas Love Peacock's *The Four Ages of Poetry*. In this work, Peacock situates himself in the literal-truth paradigm: "Feeling and passion are best painted in, and roused by, ornamental and figurative language; but the reason and the understanding are best addressed in the simplest and most unvarnished phrase," (Peacock 9) and hence "While the historian and the philosopher are advancing in, and accelerating, the progress of knowledge, the poet is wallowing in the rubbish of departed ignorance, and raking up the ashes of dead savages to find gewgaws and rattles for the grown babies of the age." (Peacock 15). One wonders whether his criticisms of poetry using poetical language are intentional and ironic, or unintentional and hypocritical.

⁸⁵Percy Bysshe Shelley, "A Defence of Poetry," *Peacock's Four Ages of Poetry, Shelley's Defence of Poetry, Browning's Essay on Shelley*, Ed. H.F.B. Brett-Smith (Oxford: Blackwell, 1929), 26.

⁸⁶Shelley 26.

Those who subscribe to the creative-imagination paradigm also tend to espouse comparable views about the development of language; namely, that definitions and dictionaries only follow where poets have already blazed metaphoric trails.⁸⁷ The very origin of the ongoing project of representing and reasoning about the world in language must lie in the human poetic faculty, the creative imagination. The essayist Thomas Carlyle summarized this developmental stance most aptly and succinctly: “The coldest word was once a glowing new metaphor.”⁸⁸

Although most philosophers of this period favoured the literal-truth paradigm, a few were vocal proponents of some version of the creative-imagination position. For example, Jean-Jacques Rousseau (1712–1778) can be seen to support the creative-imagination paradigm in his *Essay on the Origin of Languages*: “As man’s first motives for speaking were of the passions, his first expressions were tropes. Figurative language was the first to be born. . . At first only poetry was spoken; there was no hint of reasoning until much later.”⁸⁹ Contemporary advocates of a creative-imagination stance often recognize Giambattista Vico (1668–1744) as the father of their philosophical tradition. A rhetorician and philosopher, Vico is perhaps best known for his magnum opus *The New Science*, an ambitious work that seeks to establish “the principles of humanity,” to give an account of the development and functioning of typically non-quantifiable human practices and institutions. His method in this undertaking constitutes a “new science” insofar as it attempts to overcome deficiencies he perceived in the then-dominant Cartesian hypothetico-deductive method by acknowledging the necessary contribution of human individuals and societies to all things. His approach is to form a synthesis of philosophy and philology — thereby vitalizing the truths of the former using the certainties of the latter — and to use this “new critical art” to examine human experience.⁹⁰ Vico summarizes the end of his undertaking as follows: “Thus our Science comes to be at once a history of the ideas, the customs, the deeds of mankind. From these three we shall

⁸⁷The term “poet” is sometimes used more generally to describe anyone who invokes creative imagination, and not only those who do so in writing poetry (in the narrow sense). Shelley certainly was friendly to such a general interpretation: “Poetry, in a general sense, may be defined to be ‘the expression of the Imagination’” (Shelley 23). The justification for this is typically etymological: “poetry” derives from the Greek word *ποιεῖν*, a verb meaning to create, make, compose, prepare, write, produce, etc. (Liddell and Scott).

⁸⁸Thomas Carlyle, *Past and Present*, Ed. Chris R. Vanden Bossche (Berkeley: U of California P, 2005), 130.

⁸⁹Quoted in Johnson, “Introduction” 15.

⁹⁰Timothy Costelloe, “Giambattista Vico,” *Stanford Encyclopedia of Philosophy*, 11 Mar. 2009.

derive the principles of the history of human nature, which we shall show to be the principles of universal history, which principles it seems hitherto to have lacked.”⁹¹

In the first pages of *The New Science*, Vico examines the history of language as centrally important in the history of humanity. Vico’s assumptions lead him to conclude that “the principle of the origins both of languages and of letters lies in the fact that the early gentile people, by a demonstrated necessity of nature, were poets who spoke in poetic characters. This discovery...is the master key of this [new] Science.”⁹² That is, the contribution that humans made to the origins of language must have ultimately been a work of the creative imagination. In particular, according to Vico, metaphor played an important role in the origins of language and therefore in the history of humanity:

The most luminous and therefore the most necessary and frequent [of the first tropes] is metaphor. It is most praised when it gives sense and passion to insensate things, in accordance with the metaphysics above discussed, by which the first poets attributed to bodies the being of animate substances, with capacities measured by their own, namely sense and passion, and in this way made fables of them. Thus every metaphor so formed is a fable in brief. This gives a basis for judging the time when metaphors made their appearance in the languages. All the metaphors conveyed by likenesses taken from bodies to signify the operations of abstract minds must date from times when philosophies were taking shape. The proof of this is that in every language the terms needed for the refined arts and recondite sciences are of rustic origin.⁹³

For Vico, metaphor was the characteristic trope of the second of the three ages of the world, the Heroic age. Metaphor allowed poets like Homer to extend the simple representative hieroglyphs of the first age beyond their natural immediate referents. It thus allowed language to address entities and activities of a less corporeal nature: “vulgar Latin... has formed almost all its words by metaphors drawn from natural objects according to their natural properties or sensible effects. And in general metaphor makes up the great body of the language among all nations.”⁹⁴ There are a couple of important points to take from Vico. First, his

⁹¹Giambattista Vico, *The New Science of Giambattista Vico*, Trans. Thomas Bergin and Max Fisch (Garden City, New York: Doubleday Anchor, 1961), §368, 73. It is important to note the motivation and nature of Vico’s book because of their drastic departure from the Cartesian project to make all thought approach the standard set by deductive geometric proof based upon clear and distinct axioms; the Cartesian “geometric” approach, the literal-truth paradigm, and the notion that mathematics is devoid of metaphor are all intimately interconnected.

⁹²Vico, §34, 5.

⁹³Vico, §404, 87–8.

⁹⁴Vico, §444, 104.

injection of etymology and history into philosophy helps vindicate the approach taken in this chapter. More generally, his approach suggests that a theory of metaphor ought to include some considerations of the development of language. And second, Vico is possibly the first philosopher to note that a large proportion of words in contemporary use involve metaphor in their etymological roots. This is a refreshing revelation after the hypocrisy of Hobbes. It must be noted, however, that Vico's theory suggests that all words eventually find their ground in the signs based in natural relation and resemblance that comprised the language of the first age; the extent to which Vico considered these first signs to be metaphorical is unclear. One final pre-twentieth century philosopher warrants consideration for deviating from the literal-truth paradigm even more radically than Vico.

Friedrich Nietzsche (1844–1900) presents a view of metaphor in one of his earliest works, *On Truth and Lies in an Extra-Moral Sense*.⁹⁵ In this manuscript, Nietzsche questions why humans desire truth, especially given how deeply deception is connected to biology; for example, he explains how deception forms a part of several successful survival strategies. He concludes that the drive for truth originates in the fact that humans are social animals and therefore need a peace agreement to prevent descent into a Hobbesian state of nature. However, this origin has significant implications for Nietzsche's understanding of truth:

What then is truth? a mobile army of metaphors, metonyms, anthropomorphisms, in short, a sum of human relations which were poetically and rhetorically heightened, transferred, and adorned, and after long use seem solid, canonical, and binding to a nation. Truths are illusions about which it has been forgotten that they *are* illusions, worn-out metaphors without sensory impact, coins which have lost their image and now can be used only as metal, and no longer as coins.⁹⁶

Nietzsche's view is strongly antithetical to the literal-truth paradigm, in that it posits that all words and signs are necessarily metaphorical because things-in-themselves, and hence capital-T Truths, are transcendently beyond our reach. This perspective is similar to the Vichean in emphasizing metaphor and its constitutive role in language, though Nietzsche's stance is definitively more extreme: he claims that "every concept originates through the equation

⁹⁵Though Nietzsche apparently later rejected the Romanticism of his early writings, this paper is worthy of consideration as part of this context chapter due to its unique outlook on metaphor.

⁹⁶Friedrich Nietzsche, "On Truth and Lying in an Extra-Moral Sense," *Friedrich Nietzsche on Rhetoric and Language*, Ed. and trans. Sander L. Gilman, Carole Blair, and David J. Parent (New York: Oxford, 1989), 250.

of the dissimilar,” and concludes that even the uptake and integration of neural stimuli constitutes metaphor.⁹⁷ Interestingly, Nietzsche further exposes his Kantian assumptions by claiming that the intuitions of space and time (as discussed at length in the *Critique of Pure Reason*) and their mathematical regularity are presupposed in the creation of all metaphors, and thus transitively in concepts, the well-worn residues of metaphors. Nietzsche seems to suggest, then, that at least some of mathematics might not be metaphorical insofar as it is part of, or emergent from, the regularity-providing intuitions that are a precondition for the fabrication of metaphors. Indeed, mathematics seems to be part of this regularity. Despite some confusing Kantian-flavoured remarks about mathematics and a few incorrect or obsolete biological assumptions, Nietzsche’s radical claims about metaphor are ahead of their time insofar as they seem to anticipate several important features of the Lakovian theory of a century later that is the focus of chapter 3.

The relatively small motley of poets, artists, and philosophers who were the early supporters of the creative-imagination paradigm made some important contributions to the philosophy of metaphor that are reflected in many of the theories of the twentieth century. Foremost among these contributions is the idea that human creativity plays an essential role in the origin and development of language; this supposition entails that metaphor is not merely an afterthought — a disposable garnish on the enchilada of literality — but rather precedes and makes an etymological contribution to many now-common words. This line of thought reaches its apex in Nietzsche’s approach which presciently considers the development of language as part of the phylogenetic development of the human species. It is in the developmentally and biologically oriented writings of the creative-imagination theorists that the notion of dead metaphor first arises in this historical summary; though the phrase is not explicitly invoked, the idea is clearly present. At first glance, the metaphor of “dead metaphor” seems an easy way to understand the developmental ideas of the creative-imagination theorists: novel metaphoric phrases gain popularity and, through time and use, lose their metaphoricity. The idea of dead metaphor is not consistent with extreme versions of the literal-truth paradigm (as it acknowledges metaphor as more than merely derivative), but does seem to fit with more moderate claims regarding the primacy of literal language.

⁹⁷Nietzsche 249.

Because of its apparent aptness and its consistency with a wide variety of theories, many subsequent authors have used the idea of “dead metaphor” without providing significant explication. Though the creative-imagination theorists provided some important ideas to the philosophy of metaphor, many people shy away from adopting the paradigm wholesale, probably because of a worry that it weakens the notion of truth. As such, the literal-truth paradigm remained dominant into the twentieth century.

2.3 Twentieth Century

The powerful logical positivist movement that occurred in philosophy during the first half of the twentieth century exhibited extreme devotion to the literal-truth paradigm. Members of this movement emphasized empirical verifiability of statements as crucial to meaningfulness, and this caused them to be suspicious or outright dismissive of statements that failed to fully live up to this criterion. Famously, logical positivists were highly critical of the statements of theology and ethics, as well as positive universal claims.⁹⁸ In his “Lecture on Ethics,” Wittgenstein shows that logical positivism was also dismissive of metaphor:

if I can describe a fact by means of a simile I must also be able to drop the simile and to describe the facts without it. Now in [ethical and religious language] as soon as we try to drop the simile and state the facts which stand behind it, we find that there are no such facts. And so, what at first appeared to be simile now seems to be mere nonsense.⁹⁹

Though this passage is explicitly examining the use of simile in theology and ethics, it betrays certain presuppositions and biases regarding simile and, arguably therefore, metaphor. What appears to be a simile must either be merely a garnished version of a factual statement that can be reached through paraphrase, or else be a pseudo-simile, an empty lump of garnish deceptively masquerading as a paraphrasable simile, that is, “mere nonsense,” devoid of meaning. Insofar as this passage is representative of the logical positivist movement, it exposes them as being as negative regarding metaphor as the early modern empiricists. Thus, though the philosophy of language gained new momentum in the early twentieth

⁹⁸John Skorupski, “Later Empiricism and Logical Positivism,” *The Oxford Handbook of Philosophy of Mathematics and Logic*, Ed. Stewart Shapiro (New York: Oxford UP, 2005), 65–9.

⁹⁹Ludwig Wittgenstein, “Lecture on Ethics,” *The Philosophical Review* 74.1 (1965): 10.

century, most philosophers remained orthodox advocates of the literal-truth paradigm and did not advance new theories of metaphor. I am aware of only one significant counterexample to this trend.¹⁰⁰

I.A. Richards (1893–1979) was a philosopher and a rhetorician who was influential within the study of English literature. Richards' 1936 book *The Philosophy of Rhetoric* contains a chapter on metaphor, the thesis of which is that some of the major assumptions of the literal-truth paradigm date back to Aristotle and have prevented the development of a suitable theory of metaphor. Insofar as Richards denies that metaphor is deviant and instead claims that "[the fact] that metaphor is the omnipresent principle of language can be shown by mere observation,"¹⁰¹ he reveals himself as an advocate of the creative-imagination paradigm. This classification is further substantiated by his recognition that Shelley and historians of language were among the exceptional few who saw past the problematic tradition. There are two significant ways in which Richards goes beyond his forebears in developing a significant theory of metaphor. First, Richards notes that "when we use a metaphor we have two thoughts of different things active together and supported by a single word, or phrase, whose meaning is a resultant of their interaction."¹⁰² A claim rather like this is already made by Aristotle; what makes Richards' assessment significant is that he goes on to make the observation that "One of the oddest of the many odd things about the whole topic is that we have no agreed distinguishing terms for these two halves of a metaphor — in spite of the immense convenience, almost the necessity, of such terms if we are to make any analyses without confusion."¹⁰³ Richards proposes the adoption of some technical jargon to help eliminate some of the vagueness and ambiguity: the *tenor* of a metaphor is the underlying idea or subject that is conveyed by means of the *vehicle*. Clumsily, the tenor might be described as "the original idea" while the vehicle is "the borrowed idea."¹⁰⁴ Richards concludes that the adoption of this terminology serves to circumvent the problematic imposition of imagery terminology,

¹⁰⁰It should be remarked that my present familiarity with philosophical literature of this period in no way precludes the existence of other relevant papers and authors. As was noted at the beginning of this chapter, this historical survey is necessarily a selective sampling of the relevant literature, and one of the unfortunate but necessary constraints on this process is the limits of my knowledge. Please forgive any glaring omissions.

¹⁰¹I.A. Richards, "Lecture V: Metaphor," *The Philosophy of Rhetoric* (Oxford: Oxford UP, 1936), 50.

¹⁰²Richards 51.

¹⁰³Richards 52.

¹⁰⁴Richards 52.

and also allows the insight that “in many of the most important uses of metaphor, the co-presence of the vehicle and tenor results in a meaning (to be clearly distinguished from the tenor) which is not attainable without their interaction.”¹⁰⁵ Richards goes on to note that it would be worthwhile to consider interactions between tenor and vehicle based in relations other than resemblance, including relations of disparity.¹⁰⁶ Thus, Richards’ careful dissection of metaphor gives a deeper insight into what it might be and how it might function. The second revelation of Richards’ lecture is that “[t]hought is metaphoric, and proceeds by comparison, and the metaphors of language derive therefrom. To improve the theory of metaphor we must remember this.”¹⁰⁷ The idea that metaphoric sentences are merely instantiated symptoms of underlying metaphoric thought is a powerful one that is later used efficaciously by Lakoff. Richards’ account is more provocative than definitively detailed, but his novel insights nonetheless constitute an important contribution to the philosophy of metaphor.

The next significant landmark in the history of the philosophy of metaphor is Max Black’s (1909–1988) seminal 1955 paper “Metaphor.” Though Black was clearly influenced by Richards in the writing of this paper, it is “Metaphor” and not *The Philosophy of Rhetoric* that marks the beginning of a new age in the philosophy of metaphor. There seem to be three main reasons for this. First, while Richards was a rhetorician and literary theorist first and foremost, Black was an eminent and respected philosopher of language at the time “Metaphor” was published.¹⁰⁸ This likely resulted in his work having a wider exposure among philosophers and thus more apt to provoke philosophical responses. Second, the nearly twenty-year gap between Richards’ and Black’s publications saw some developments in the philosophy of language (for example, the emergence of ordinary language philosophy) that seem to have rendered the 1950s a more receptive time for the types of ideas presented in these papers. And third, Black’s treatment of metaphor is nicely systematic, a paragon instance of a philosophical paper. While Black published other works dealing with the subject

¹⁰⁵Richards 55.

¹⁰⁶Richards 59–60.

¹⁰⁷Richards 51; emphasis his.

¹⁰⁸A relevant but tangential fact: Max Black possessed a B.A. in mathematics from Cambridge and was employed as a math lecturer previous to receiving his professorship in the philosophy of language (O’Connor and Robertson).

of metaphor, I shall focus on his views presented in “Metaphor.”¹⁰⁹

As noted above, one of the key merits of Black’s paper is its systematicity. He opens the paper by noting the widespread acceptance by philosophers of the commandment “Thou shalt not commit metaphor” and suggesting this has contributed to a dearth of adequate theories. Black then suggests a list of questions that any adequate theory of metaphor should provide answers to:

How do we recognize a case of metaphor?
Are there any criteria for the detection of metaphors?
Can metaphors be translated into literal expressions?
Is metaphor properly regarded as a decoration upon “plain sense”?
What are the relations between metaphor and simile?
In what sense, if any, is a metaphor “creative”?
What is the point of using a metaphor?¹¹⁰

By making these questions explicit, Black provided important structure both for his own paper and for subsequent authors. In the wake of these questions, Black presents a collection of sentences that he hopes will be unmistakably recognized as examples of metaphor. Though the sentiment is admirable, this is one of the points where his theory seems weakest. Whereas Black claims that “The rules of our language determine that some expressions must count as metaphors” regardless of the “occasions on which the expressions are used,” later pragmatic theories of metaphor will convincingly argue that the textual context a sentence is uttered in plays a key role in the interpretation of that sentence, including whether it is interpreted metaphorically or literally.¹¹¹ Black’s examples consist only of isolated sentences and therefore are devoid of textual context, leaving them potentially ambiguous.¹¹² One might take this conclusion further and claim that even when extended textual passages are quoted as examples in philosophical papers on metaphor, the transposition into a new context necessarily has an impact on interpretation; thus, quoted metaphors are like animals in captivity: one must not assume that their “behaviour” is fully natural. Another

¹⁰⁹For more of Black’s thoughts about metaphor, see his “More About Metaphor” (1979) and *Models and Metaphors* (1962).

¹¹⁰Max Black, “Metaphor,” *Proc. of Aristotelian Society* 55 (1954-55): 64.

¹¹¹Black, “Metaphor” 66.

¹¹²Note that the metaphoricality of Black’s first example seems less compellingly unmistakable when it is given some context: “The chairman plowed through the discussion. Nine were killed by the madman’s rusty implement.”

potential difficulty surrounds the presentation of explicit examples. Black seems to suggest elsewhere that the concept of METAPHOR, like that of GAME, is connected by Wittgensteinian family resemblance.¹¹³ An intuition very much like this one is the primary reason for the conspicuous absence of examples from this chapter so far. In any case where a complicated category is being considered, there is always a danger that examples given will be mistakenly taken as paradigmatically definitive of the category by the reader. That is, whether a robin, a penguin, or an ostrich is presented as an example of a bird, it is likely to introduce a bias that skews the reader's understanding of the category BIRD. To be fair, Black has provided a wide range of examples to help fend off this concern, something that many other authors fail to do. However, there is a key difference between birds and metaphors that is relevant here: whereas new birds may be *discovered* that challenge the understanding of the category emerging from the presentation of multiple examples, novel metaphors may be explicitly *constructed* to serve this function. Thus, the nature of metaphor seems to make it ill suited for explication through even an extensive array of examples. The bird example was chosen specifically to both explain the introduction of bias and to illustrate it on a meta-level by introducing a bias regarding family-resemblance concepts. The reader is urged to remain cognizant of the warnings of this paragraph whenever a list of example metaphors is encountered in the literature.

Based on his provided examples, Black makes the following observation: “In general, when we speak of a relatively simple metaphor, we are referring to a sentence or another expression, in which *some* words are used metaphorically, while the remainder are used non-metaphorically. An attempt to construct an entire sentence of words that are used metaphorically results in a proverb, an allegory, or a riddle.”¹¹⁴ He then implements some technical jargon to reinforce the above observation: the *focus* of a metaphorical sentence is defined as the word or phrase being used metaphorically within a non-metaphoric *frame*. While this distinction is reminiscent of Richards', it is important to note that the tenor/vehicle

¹¹³Andrew Ortony, “Metaphor: a multidisciplinary problem,” *Metaphor and Thought*, Ed. Andrew Ortony, 1st Ed. (New York: Cambridge UP, 1979), 5.

¹¹⁴Black, “Metaphor” 65.

distinction is not identical to the focus/frame distinction.¹¹⁵ Black's quote also seems to pay homage to Aristotle's suggestion that a balance between metaphoric and non-metaphoric speech is desirable, with an excess of metaphor giving rise to riddles. A couple of additional general claims about metaphor that Black makes are relevant to this discussion. Black argues that metaphor is not a syntactic phenomenon — the recognition and interpretation of metaphorical sentences seems to be independent of grammar — but rather has to do with meanings of sentences. His evidence for this claim comes from his belief that the *very same* metaphor can be encoded in multiple languages.¹¹⁶ He concludes that metaphor is therefore within the domain of semantics, though he acknowledges a pragmatic dimension that “may be the one most deserving of attention”: the intended gravity of the focus — the “depth” of the metaphor — seems to be strongly context dependent.¹¹⁷ Unfortunately, Black spends little time pursuing this observation. Instead, the remainder of the paper is dedicated to the discussion of three competing views of metaphor.

Black rejects what he labels substitution and comparison views of metaphor before presenting his own interaction view. Substitution views of metaphor hold that every instance of a metaphorical sentence is a substitute for some equivalent literal expression.¹¹⁸ Such views have dominated throughout the history of the subject, as has been noted above, and lead to the condemnation of metaphor as distracting and potentially confusing ornamentation. If such a view were true, what purposes could metaphor possibly serve? Black suggests two possibilities: metaphor could be employed catachrestically when no literal equivalent exists or its use could be merely stylistic.¹¹⁹ Though the nomenclature was not invoked, substitution views were criticized earlier in this chapter for being hypocritical and contradicting linguistic evidence, and Black likewise rejects them, moving on to consider comparison views. Any comparison view of metaphor can be seen as a specific variety of substitution view, in

¹¹⁵That is, while focus and frame are both defined as parts of a sentence, vehicle and tenor seem more akin to sentence and speaker meaning, respectively.

¹¹⁶Black, “Metaphor” 66.

¹¹⁷Black, “Metaphor” 67.

¹¹⁸Black, “Metaphor” 68.

¹¹⁹Black notes that “[i]t is the fate of catachresis to disappear when successful” (“Metaphor” 69); that is, such usages tend to be conventionalized into the lexicon if a gap indeed exists. It is the suggestion that all metaphors are catachrestic that Donald Davidson objects to when he says “to make a metaphor is to murder it” (“What Metaphors Mean” 437). One arguably positive stylistic use of metaphor outside of literature is in euphemism, the use of inoffensive terms to communicate potentially offensive ideas.

that “it holds that the metaphorical statement might be replaced by an equivalent literal *comparison*”;¹²⁰ that is, comparison views postulate that metaphors are elliptical similes. Not only do comparison views have to answer the same criticisms as substitution views (why bother to express the simile indirectly?), they are subject to a further difficulty; as Black puts it, they suffer from “a vagueness that borders upon vacuity” because of the unconstrained breadth of the likeness relation: any thing is like any other thing in various, and usually numerous, ways.¹²¹ Thus, saying that a metaphor is a comparison without significant further explication of the notion of comparison does little to no explanatory work. Black contends that his interaction theory overcomes both the redundancy and vacuity defects.

Black introduces his interaction theory with the following gloss: “when we use a metaphor we have two thoughts of different things active together and supported by a single word, or phrase, whose meaning is a resultant of their interaction.”¹²² If the meaning of such metaphors emerge through such an interaction, then it follows that neither of the two interacting components are redundant and, therefore, eliminable through substitution.¹²³ Another interesting consequence of the positing of such emergent meanings is that metaphors seem to be able to *create* similarities in some cases rather than being necessarily based in pre-existing similarity.¹²⁴ The metaphoric interaction is supposed to proceed as follows: the frame of a metaphor forms a context for the focus that extends its meaning by inviting the audience to see one thing as another. Black claims that talk about some thing evokes a “system of associated commonplaces,” a collection of accurate attributions, half-truths, stereotypes, and

¹²⁰Black, “Metaphor” 71.

¹²¹“Metaphor” 71.

¹²²Black, “Metaphor” 72.

¹²³Even if one accepts Black’s interaction theory as correct, it seems that some particularly shallow metaphors could be successfully eliminated through substitution. In his later work, “More About Metaphor,” Black implements jargon in saying that ineliminable and unparaphrasable metaphors are more *emphatic* (26). Towards the end of “Metaphor,” Black puts forward the suggestion that metaphors might be taxonomically classified as substitution, comparison, or interaction metaphors (78). Meaningfully following this line of thought further would seem to require the discussion of the pragmatics of metaphor that Black does not provide.

¹²⁴Given the previous paragraph’s remark about the breadth of the likeness relation, this sentence could use some clarification. Though it might be accurate to say “everything is like everything else in numerous ways,” perhaps this ought to be qualified by “in potentia.” While some likenesses or resemblances have been repeatedly recognized and thoroughly conventionalized as similarities, other metaphoric and similitive sentences seem to require a more substantial creative effort. For example, contrast “Arms are like tentacles” with “Ravens are like writing desks.” The idea that metaphors are at least sometimes creative in this way is one I will want to insist on.

other sentiments regarding that subject that are commonly held within some culture.¹²⁵ It is the systems of commonplaces associated with each of the two “subjects” of the metaphor that interact, the “subsidiary subject” organizing the “primary subject” by emphasizing certain of its aspects and de-emphasizing others. For example, in Black’s example “Man is a wolf,” “man” is the primary subject and “wolf” is both the subsidiary subject and the focus of the metaphor. The metaphor invites the audience to see men as wolves via the basic cultural understanding of wolfhood, and thus serves to extend the meaning of “wolf” to have a sense that is applicable to “man” while maintaining its original senses.¹²⁶ Given this brief sketch of Black’s interaction theory, a few important remarks are required. First, Black’s view does not explain which precise aspects will be (de-)emphasized or the magnitude of this (de-)emphasis; as previously noted, further progress in this direction would require some discussion of the pragmatics of metaphor. A second related point is that in some cases, an author may contrive a textual context that is meant to augment or override the system of associated commonplaces: “in a poem, or a piece of sustained prose, the writer can establish a novel pattern of implications for the literal uses of the key expressions, prior to using them as vehicles for his metaphors.”¹²⁷ Thirdly, Black contends that interaction metaphors have a bidirectional symmetry: “If to call a man a wolf is to put him in a special light, we must not forget that the metaphor makes the wolf seem more human than he otherwise would.”¹²⁸ Finally, and perhaps most significantly, Black notes that the interaction view as he has presented it employs metaphor in its discussion of metaphor (metaphor is a filter, metaphor is a projection, etc.). While such an employment was hypocritical in Hobbes, it seems only reasonable in Black’s writing given his claim that “‘Metaphor’ is a loose word, at best, and we must beware of attributing to it stricter rules of usage than are actually found in practice”;¹²⁹ indeed, he explicitly states that he has “no quarrel with the use of metaphors (if they are good ones) in talking about metaphor. But it may be as well to use several, lest

¹²⁵Black, “Metaphor” 74.

¹²⁶Black, “Metaphor” 73–4.

¹²⁷Black, “Metaphor” 77.

¹²⁸Black, “Metaphor” 77.

¹²⁹Black, “Metaphor” 66. It is no surprise that Black is comfortable with loose concepts: his early work on vague sets and language anticipates the advent of fuzzy logic by nearly three decades (O’Connor and Robertson).

we are misled by the adventitious charms of our favourites.”¹³⁰ Hence, interaction metaphors are powerful, irreplaceable devices that can be used to great effect in organizing and communicating complex and important meanings throughout the various areas of human discourse — including philosophy. However, one must remain cognizant of the potential dangers of using such metaphors: they can depend on — and therefore perpetuate — false, harmful stereotypes, and if an interaction metaphor becomes an exclusively dominant way of seeing some thing, it can be problematically limiting insofar as such metaphors tend to obfuscate some important aspects of their primary subject in order to illuminate others.

Black’s interaction view is an elaboration and refinement of ideas found in Richards, and provocatively defies the traditional views by suggesting that certain strong metaphors are ineliminable and may be legitimately used in philosophical discourse. The question remains whether Black’s theory belongs to either of the two paradigms posited in this chapter, as it seems to fall somewhere between the “Metaphor is eliminable ornamentation” and “All language is metaphoric” extremes. On the one hand, Black’s theory emphasizes the ineliminability and creativity of a class of metaphors; on the other hand, his theory does seem to appeal to an underlying notion of literality in its understanding of metaphor at times, insofar as literality plays some role in the system of associated commonplaces. Perhaps the best answer to this question is that Black’s theory does not seem to be clearly contained in either of these paradigms, but rather served to restructure the way metaphor was viewed. Rather than simply characterizing metaphor as eliminable or essential, the relative deluge of papers written in Black’s wake tend to focus on identifying or classifying metaphors, or on explaining how they work and when they are useful, or on considering the cognitive status of metaphor.¹³¹ Thus, the literal-truth versus creative-imagination classification seems ill equipped to handle the multidimensionality of metaphor studies in the later half of the twentieth century. Fortunately, Black’s paper seems to recommend some alternative classification schemes. One such scheme is presented explicitly by Black himself: theories of metaphor could be classified as substitution, comparison, or interaction views. Another possibility would be to attempt to classify theories based on their answers to the list of questions posed by Black

¹³⁰Black, “Metaphor” 73.

¹³¹Johnson, “Introduction” 20.

in “Metaphor.” However, it is the distinction between pragmatic and semantic theories that seems primarily relevant in classifying the theories of metaphor in the period between Black and Lakoff. Most of the leading theories from this time period are pragmatic, a trend that seems natural given Black’s provocative suggestion that the pragmatics of metaphor might be of primary import despite his own semantic stance. However, it is worth spending a little time considering semantic theories before moving on to the predominant pragmatic trend.

Not all views of metaphor after Black are pragmatic. Though Black is the first author to identify his position as a semantic one, earlier substitution and comparison views are also typically semantic; thus, advocates of traditional theories tend to be semantic theorists. One such was Paul Henle, whose contribution to the philosophy of metaphor was to update some of the traditional claims about metaphor using C.S. Peirce’s account of symbols and icons.¹³² However, in the post-Black period, most semantic theories of metaphor bore a closer resemblance to his interaction theory than to traditional views. Michiel Leezenberg includes Black’s interaction theory in a class of views he refers to as *descriptivist* that also includes those of Monroe Beardsley and Nelson Goodman.¹³³ Descriptivist views of metaphor are united by the idea that the interpretation of a metaphor is “guided by the descriptive information associated with an expression” rather than being a matter of a similarity or some other relation between the referents of a metaphorical expression.¹³⁴ Views like those of Cicero and Henle are classified as *referentialist* because they satisfy this latter criterion.¹³⁵ The number of referentialists dwindled shortly after the advent of descriptivist theories and their criticisms of various tenets of referentialism.¹³⁶ Similarly, many semantic descriptivists converted to a pragmatic approach once it had been established as a genuine alternative; the later writings of Max Black suggest he was among said converts. While clear examples of both referentialist and descriptivist semantic theories exist, some important authors defy classification.

¹³²Johnson, “Introduction” 25.

¹³³Leezenberg 10–1.

¹³⁴Leezenberg 11.

¹³⁵Leezenberg 71. Leezenberg situates this classification scheme orthogonal to the semantic/pragmatic divide.

¹³⁶Though I will not discuss their views here, it should be noted that referentialist viewpoints did not die out altogether post-Black. Two later referentialists mentioned by Leezenberg are Mooij and Fogelin (74).

Paul Ricoeur's *La métaphore vive* (translated into English as *The Rule of Metaphor*) is among the most voluminous works on metaphor to date. Ricoeur's work repeatedly identifies itself as semantic, but his suggestion that he will reconcile substitution and interaction by appealing to hermeneutics makes it somewhat unclear whether his view ought to be classified as referentialist, descriptivist, or as belonging to some heretofore unnamed third category.¹³⁷ Due to the scope of Ricoeur's work and its connexions to the phenomenological tradition, it is not feasible to summarize it here, even in brief; however, two especially relevant passages must be mentioned before moving on. First, Ricoeur claims that the most important theme of his work is that "metaphor is the rhetorical process by which discourse unleashes the power that certain fictions have to redescribe reality."¹³⁸ This alleged connexion between metaphor and fiction provides motivation for the discussion of mathematical fictionalism in chapter 4. Second, "the 'place' of metaphor, its most intimate and ultimate abode, is... the copula of the verb *to be*. The metaphorical 'is' at once signifies both 'is not' and 'is like.'"¹³⁹ The combination of these two significations imbues the copula with the sense of *being-as*.¹⁴⁰ Although it is not immediately clear how this remark can be true of metaphorical expressions that do not contain a copula, it is resonant with the standard TARGET IS SOURCE notation for conceptual metaphors used by conceptual metaphor theorists. Having dispensed with semantic views for the time being, I now consider pragmatic accounts of metaphor.

Whereas semantic views of metaphor hold that there is some kind of alteration or bifurcation of the meanings of some words in every metaphorical expression, pragmatic views hold word meanings constant and instead suggest metaphorical interpretation occurs apart from meanings, at the level of the conversational participants. Many supporters of a pragmatic view of metaphor adopt an approach rooted in H.P. Grice's theory of conversational implicature.¹⁴¹ In "Logic and Conversation," Grice makes the observation that there is an apparent divergence in meaning between the connectives of formal logic and the words in English typically associated with them; anyone who has taken a logic course is familiar with

¹³⁷Paul Ricoeur, *The Rule of Metaphor*, Trans. Robert Czerny (Toronto: U Toronto P, 1975), 66.

¹³⁸Ricoeur 7.

¹³⁹Ricoeur 7.

¹⁴⁰Ricoeur 257.

¹⁴¹Some notable authors adopting such a viewpoint include Merrie Bergmann, Stephen Levinson, A.P. Martinich, and, of course, H.P. Grice.

the difficulties and confusion that can arise from the differences between understandings of the symbolic formal connective \vee and the English word “or,” for example.¹⁴² Motivated by the observation that what is communicated often differs from or extends beyond what is explicitly said, Grice jargonizes the verb “implicate” to describe such communicative acts. By distinguishing between the conventional meanings of an utterance and its implicata, Grice bypasses the quagmire of controversy and disagreement that comes with the positing of multiple simultaneous divergent meanings, allowing for the construction of a positive theory. Though there are several varieties of implicature, Grice argues that metaphor is a variety of non-conventional, conversational implicature.¹⁴³

There is a cluster of extralinguistic conventions governing conversations insofar as they are rational cooperative communicative exchanges. Principal among these is the Cooperative Principle: “Make your conversational contribution such as it is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged.”¹⁴⁴ This principle works in conjunction with four categories of maxims which Martinich helpfully integrates into the following supermaxims:

Quality: Do not participate in a speech act unless you satisfy all the conditions for its nondefective performance.

Quantity: Make your speech act as strong as appropriate but not stronger than appropriate.

Relation: Make your contribution to the conversation one that ties in with the general course of the conversation.

Manner: Make your contribution brief, clear, orderly, and unambiguous.¹⁴⁵

¹⁴²H.P. Grice, “Logic and Conversation,” *Syntax and Semantics*, Eds. Peter Cole and Jerry L. Morgan (New York: Academic Press, 1975), 165. “Meaning” here must refer to non-natural meaning, the kind of meaning that is created by humans and arises through a reciprocal recognition between speaker and audience that empowers the intention behind an utterance. Talk of intentions and non-natural meanings is conspicuously absent from “Logic and Conversation.” For more details on non-natural meaning, see Grice’s “Meaning” (1957).

¹⁴³Conventional implicatures are those which arise directly from the conventional meanings of words and the rules of language. For example, the inference from the utterance “Bob and Doug drank beer” to the conclusion “Bob drank beer” is a case of conventional implicature. Non-conventional implicatures depend on institutions or other factors beyond the standard conventions of language (Grice, “Logic and Conversation” 167).

¹⁴⁴Grice, “Logic and Conversation” 167.

¹⁴⁵A.P. Martinich, “A Theory of Fiction,” *Philosophy and Literature* 25 (2001): 100. Martinich’s formulation of the supermaxims is superior to Grice’s categories in being more concise and simultaneously more general insofar as they cover speech acts other than assertions.

It is through the expectation that conversation participants are adhering to the Cooperative Principle and the four supermaxims that conversational implicatures arise. In some cases, a conversation partner may make a contribution that obviously and ostentatiously fails to satisfy one of the supermaxims; the obviousness of the failure suggests that the speaker intended for the failure to occur and also intended for the audience to notice the failure. Thus, the obviousness of the failure suggests that the speaker is not intending to deceive and can be assumed to be otherwise conforming to the conversational maxims; therefore, the audience can assume that the ostentatious failure is relevant and thereby seek out implicatures that make sense of the contribution. When such an ostentatious failure occurs, Grice says that a maxim has been flouted, and when a flouting is used to generate an implicature, he says that a maxim has been exploited.¹⁴⁶ Paraphrased into Martinich's terminology, Grice's claim is that metaphorical utterances flout the supermaxim of Quality by characteristically being categorically false and that the implicatures are often grounded in resemblances.¹⁴⁷ He also notes that metaphors are sometimes used ironically and therefore concludes that implicature is layerable.¹⁴⁸ Grice's discussion of metaphor is rather scant, but A.P. Martinich elaborates on how metaphors implicate.¹⁴⁹

Martinich explicitly notes that his Gricean theory of metaphor is "blatantly pragmatic" insofar as whether a sentence flouts the supermaxim of Quality usually depends on its context of use; thus, both the recognition and interpretation of metaphor are strongly context dependent for Martinich.¹⁵⁰ The following is a summary of Martinich's account of how implicature occurs for a typical metaphor:

1. The speaker utters "Jeanne is a fox."
2. The hearer recognizes that a) due to context, the utterance is literally false (since

¹⁴⁶Grice, "Logic and Conversation" 170.

¹⁴⁷Aristotle seems to suggest the rudiments of a mechanism like implicature in the *Poetics*: "whenever also a word seems to imply some contradiction, it is necessary to reflect how many ways there may be of understanding it in the passage in question" (Trans. Bywater 1461a32-34).

¹⁴⁸Grice, "Logic and Conversation" 172.

¹⁴⁹While Martinich's theory seems mostly consistent with Grice's, there are a few noteworthy differences: a) Grice used the less worrisome phrase "conventional meaning" in the types of situations where Martinich uses the more inflammatory "literal meaning"; b) The few sentences Grice provided about metaphorical implicature suggest that his understanding might be better classified as a comparison view, while Martinich's elaboration seems to be an interaction view.

¹⁵⁰Martinich, "A Theory for Metaphor," *Journal of Literary Semantics* 13 (1984): 457.

Jeanne is a person and hence not a fox); b) such an obvious falsehood must be a case of flouting the supermaxim of Quality; and, therefore, c) the speaker's utterance must have been a making-as-if-to-say intended to communicate by way of implicature.¹⁵¹

3. The hearer compiles a list of the various distinctive characteristics she attributes to foxes that seem relevant within the overall context (including considerations of the participants in the conversation, the histories of the participants and their shared history, the culture they live in, the setting of the conversation, the other communicative exchanges in the conversation, etc). Note that perceived salient characteristics need not be correctly attributable to the thing in question, but merely shared by the speaker and hearer for the communication to come off properly. Due to context dependence and the possible detachment from fact, a salience may be born, appealed to, and forgotten within the span of a single conversation. The idea of salient characteristics bears much similarity to Black's "system of associated commonplaces."
4. Using the original utterance and the list of salient characteristics, the hearer constructs an argument:

Jeanne is a fox.
A fox is sly, or cunning, or . . .
Therefore, Jeanne is sly, or cunning, or . . .

The conclusion of this argument is the implicatum of the metaphorical utterance.¹⁵²

Martinich notes that these implicating arguments support an interaction view of metaphor (the second premise of the syllogism is the locus of interaction), and alleges that its indeterminate disjunctions conflict with the literal paraphraseability supposition of comparison views. Later in the article, Martinich observes that, rarely, nonstandard metaphors occur that are literally true; one example might be "Jeanne is an animal." Such metaphors require an additional layer of interpretation: by saying something patently obvious and therefore

¹⁵¹At the beginning of his article, Martinich distinguishes between saying-that and making-as-if-to-say to eliminate an ambiguity he observes in Grice's writing; making-as-if-to-say occurs when an utterance is spoken in order to generate an implicature without committing to the meaning of the utterance. Martinich argues that Merrie Bergmann's account of metaphor is defective insofar as it conflates these two notions ("A Theory for Metaphor" 448).

¹⁵²Martinich, "A Theory for Metaphor" 450-2.

redundant (humans *are* animals, after all), they flout the supermaxim of Quantity, which prompts the audience to suppose they are false, allowing interpretation to proceed as in the standard case.¹⁵³ It is worth noting that Gricean theories of metaphor uphold the literal-truth paradigm to the extent that an utterance that flouts the Quality maxim necessarily deviates from the literal. Considering the less explicitly Gricean theory presented by John Searle will help bring further definition to the pragmatic understanding of metaphor.¹⁵⁴

Searle's account of metaphor is an offshoot of his general theory of speech acts. For Searle, "the main problem of metaphor is to explain how speaker meaning and sentence meaning are different and how they are, nevertheless, related."¹⁵⁵ Searle's distinction between literal sentence meaning and speaker's utterance meaning is somewhat analogous to the Gricean distinction between what a sentence says and what it implicates, and his view is similarly pragmatic as it explicitly asserts that "[m]etaphorical meaning is always speaker's utterance meaning."¹⁵⁶ For Searle, the first stage in metaphor interpretation is the identification of metaphor, so he owes an explanation of how metaphorical utterances are distinguished from both literal utterances and non-literal, non-metaphoric phenomena such as irony and indirect speech acts.¹⁵⁷ Admirably, Searle recognizes that the recognition task requires a characterization of literal utterances and notes that most authors fail to address or even to recognize this "extremely difficult, complex, and subtle problem."¹⁵⁸ There are three features of literal utterance that Searle contends are relevant to his account of metaphorical utterance. First,

¹⁵³Martinich, "A Theory for Metaphor" 454.

¹⁵⁴While I believe Searle would agree with me that his theory is not Gricean, Elisabeth Camp and other authors have used his view as the prime example of a Gricean theory (Camp and Reimer 849). This taxonomic controversy does not warrant further attention.

¹⁵⁵John Searle, "Metaphor," *Expression and Meaning* (Cambridge: Cambridge UP, 1979), 87.

¹⁵⁶Searle, "Metaphor" 77.

¹⁵⁷Recall that Grice pointed out that metaphor may be employed ironically; for example, one may say to their nemesis "You are the cream in my coffee" (Grice, "Logic and Conversation" 172). It should also be noted that though Searle limits his discussion of metaphors to the class of speech acts he calls assertions, one can also find instances of metaphor in each of his other four taxa, as well as among the indirect speech acts. Searle explains informally that an indirect speech act involves saying one thing and meaning not only that thing, but something else in addition; the canonical example is explicitly asking "Can you pass the salt?" to indirectly communicate a request to have the salt passed ("Indirect Speech Acts" 30). Building on this, an example of an utterance that is an indirect speech act and employs metaphor ironically: at a dinner party, I might ask one of my dinner companions "Can you pass the nectar of the gods?" in reference to a particularly awful bottle of wine. Savas Tsohatzidis claims that this counts as evidence against Searle's theory (Tsohatzidis 368–9).

¹⁵⁸Searle, "Metaphor" 78.

in a literal utterance, speaker meaning and sentence meaning are identical. Second, even in the most straightforwardly literal utterance, a context created by background assumptions beyond the semantic content is typically required to establish truth conditions for the sentence.¹⁵⁹ And third, any account of literal predication must rely upon an understanding of similarity.¹⁶⁰ These three observations suggest that the interpretation of literal utterances depends only on a knowledge of the rules of language and an awareness of the contexts and background assumptions that resolve the indexical and referential elements of the sentence, whereas the interpretation of metaphor must invoke some further principles; in particular, Searle notes that “[a]n analysis of metaphor must show how similarity and context play a role in metaphor different from their role in literal utterance.”¹⁶¹ Giving a precise account of what these principles are and how they allow an audience to arrive at different truth conditions for an utterance than those determined by its literal meaning is what Searle calls “the hard problem of the theory of metaphor.”¹⁶²

Given his discussion of literal language, Searle suggests that the primary strategy for metaphor detection involves the recognition of some defect arising if the utterance is taken literally, including obvious falsehood, semantic nonsense, or violations of the rules of speech acts or of conversational maxims.¹⁶³ Utterances that are literal failures are also examined as possible cases of irony (where speaker meaning is the exact opposite of sentence meaning) or indirect speech acts (where sentence meaning is a non-exhaustive component of speaker meaning). Once a potential metaphor has been detected, the hearer appeals to a set of principles to help compute potential alternative meanings. Searle contends that no single principle will suffice, and suggests a partial list of principles that can be involved when one thing calls another thing to mind, including being a necessary or contingent property

¹⁵⁹Searle discusses this point in greater detail in his “Literal Meaning,” arguing that there is no such thing as a null context for sentence interpretation (117).

¹⁶⁰Searle, “Metaphor” 81.

¹⁶¹Searle, “Metaphor” 93.

¹⁶²Searle, “Metaphor” 85.

¹⁶³Searle, “Metaphor” 105. He notes that this can not be the sole strategy for metaphor detection given the existence of what Martinich calls “nonstandard metaphors.” He also notes that another strategy depends on whether the particular author or speaker under consideration is known for being prone to metaphor use, such as in the case of a Romantic poet or a Zen master.

of, being falsely conventionally related, being similar, through a part-whole relation, etc.¹⁶⁴ Finally, the list of possible meanings is restricted to relevant ones by way of some shared strategies that include considering the textual context of the uttered sentence and comparing the predicates involved in the sentence and possible speaker meanings. While it seems Searle would hold Martinich's account to be problematically simple, Martinich contends Searle's view is problematically loose and his principles are vacuous; their theories seem practically identical in all other respects.¹⁶⁵

Searle's theory is more extensive than the above summary suggests, and there are several further relevant points that deserve consideration. Searle's Principle of Expressibility says "whatever can be meant can be said," and to the extent he holds this to be true he must give an affirmative answer to the question "Are all metaphors literally paraphrasable?"¹⁶⁶ It should be noted that this Principle refers to the infinite potential of language to represent rather than to the current lexicon of any given natural language; that is, people often use metaphor precisely when they experience a gap in the lexicon, but this does not preclude the possibility of enriching the language to include a literal expression that fills this gap.¹⁶⁷ In an extended section criticizing comparison views of metaphor, Searle makes the important observation that "Sally is a dragon" seems to be a perfectly acceptable metaphor despite the fact that dragons are fictional entities.¹⁶⁸ In "The Logical Status of Fictional Discourse," Searle notes that metaphor can be employed in both fictional and non-fictional texts, and "that what happens in fictional speech is quite different from and independent of figures of speech."¹⁶⁹ It is interesting to compare Searle's view with Martinich's claim that fiction involves a suspension of the supermaxim of Quality.¹⁷⁰ These remarks are relevant to the

¹⁶⁴The inclusion of similarity on this list is what Searle means when he says "[s]imilarity... has to do with the production and understanding of metaphor, not with its meaning" ("Metaphor" 88). Also note that by including the part-whole relation as one of his principles of metaphor, Searle explicitly classifies metonymy and synecdoche as species of metaphor.

¹⁶⁵Martinich, "A Theory for Metaphor" 456.

¹⁶⁶John Searle, *Speech Acts* (Cambridge: Cambridge UP, 1969), 88.

¹⁶⁷Searle, "Metaphor" 114.

¹⁶⁸"Metaphor" 87.

¹⁶⁹Searle, "The Logical Status of Fictional Discourse," *Expression and Meaning*, (Cambridge: Cambridge UP, 1979), 60.

¹⁷⁰Martinich, "A Theory of Fiction" 100. Martinich introduces suspension as a fourth kind of maxim contravention. It is similar to opting out, but tends to be institutional and prolonged rather than ad hoc and transient. Also, suspension does not preclude the performance of speech acts governed by the maxim under

discussion of mathematical fictionalism and its possible connexions to metaphor in chapter 4. Though pragmatic theories of metaphor were popular in the years following Black's original publication, there were other noteworthy alternatives available.

Donald Davidson's article "What Metaphors Mean" warrants attention in a survey of metaphor theories if for no other reason than the stark contrast it provides to pragmatic positions like those presented above. The atypicality of Davidson's view originates in his observation of a pervasive "central mistake" in other theories of metaphor: "the idea that a metaphor has, in addition to its literal sense or meaning, another sense or meaning."¹⁷¹ Given this observation, it is fitting that the main thesis of Davidson's paper is "metaphors mean what the words, in their most literal interpretation, mean, and nothing more."¹⁷² This is not to say that metaphors are merely ornamental or otherwise problematic; rather, Davidson explicitly explains to his readers that "[m]etaphor is a legitimate device not only in literature but in science, philosophy, and the law; it is effective in praise and abuse, praise and promotion, description and prescription."¹⁷³ How is it possible that metaphorical utterances are legitimate and useful without having a meaning over and above the typically false literal interpretation? Davidson's explanation depends on "the distinction between what words mean and what they are used to do"; insofar as his view assigns metaphor "exclusively to the domain of use," it might be classified as a pragmatic theory, though this would radically distort the traditional understanding of that taxon.¹⁷⁴ Additionally, "theory" may be a bit strong: Davidson admits elsewhere that "What Metaphors Mean" does not provide a general theory that distinguishes metaphor from other tropes and explains how metaphorical interpretations are prompted (though he does suggest that the typical patent falsity of the sentence in question may play a role).¹⁷⁵ Most of Davidson's article is dedicated to criticizing positions that have made the "central mistake." The little bit of positive theory Davidson does provide culminates in the claim that "[m]etaphor makes us see one thing as another

consideration; thus, like Searle, Martinich allows for the use of metaphor within fictional works.

¹⁷¹ "What Metaphors Mean" 435.

¹⁷² Davidson, "What Metaphors Mean" 435.

¹⁷³ Davidson, "What Metaphors Mean" 436.

¹⁷⁴ Davidson, "What Metaphors Mean" 436. Note that Davidson does not use the word "pragmatic" to refer to his own view. Camp and Reimer refer to Davidson's view as "noncognitivist" (858–9).

¹⁷⁵ Donald Davidson, "Reply to Oliver Scholz," *Reflecting Davidson: Donald Davidson Responding to an International Forum of Philosophers*, Ed. Rolf Stoecker (New York: de Gruyter, 1993), 172.

by making some literal statement that inspires or prompts the insight.”¹⁷⁶ This insight, a kind of seeing-as, is typically not propositional in nature, is potentially limitless, and requires creativity on par with that necessary for metaphor construction.¹⁷⁷ The scantness of Davidson’s positive theory seems to be an asset insofar as it better allows for the possibility of his perspective being compatible with theories of metaphor that manage to avoid his criticisms.

There are a few features of Davidson’s criticism that are worth noting. First, he says that metaphor cannot be a matter of ambiguity, insofar as “we are seldom in doubt that what we have is a metaphor.”¹⁷⁸ When genuine ambiguity or simultaneity exists regarding multiple meanings for a sentence, the resultant figure is a pun, not a metaphor.¹⁷⁹ These ambiguities between multiple meanings for a word or sentence are sometimes based in dead metaphor, when previously popular metaphorical interpretations become literally concretized. Second, a substantial portion of “What Metaphors Mean” is dedicated to a useful comparison of similes and metaphors: “We can learn much about what metaphors mean by comparing them with similes, for a simile tells us, in part, what a metaphor merely nudges us into noting.”¹⁸⁰ Davidson considers two ideas regarding the connexion between simile and metaphor. The first is the traditional “metaphors are elliptical similes” view that dates back to Cicero, the second

¹⁷⁶Davidson, “What Metaphors Mean” 445.

¹⁷⁷Davidson, “What Metaphors Mean” 444, 435. John Austin, the father of speech act theory, provides a terminological framework that may be helpful here. Austin argues that an instance of saying something can be seen as comprised of three interconnected but distinguishable acts. The *locutionary act* has phonetic (sound making), phatic (construction according to language rules), and rhetic (sense and reference imbued) components, and is the locus of meaning (Austin 95). The *illocutionary act* is the purposive component of the utterance, the intended conventional force of the utterance (Austin 109). The *perlocutionary act* is what the utterance actually accomplishes, whether intended or not (for example, uttering a threat may variously subdue, provoke a fight, incite laughter, cause disappointment, and so on) (Austin 118). In differentiating speaker and sentence *meaning*, Searle seems to emphasize the locutionary component of the metaphorical utterance; the point of the speech act is the transmission of the metaphorical speaker meaning to the listener. Davidson, on the other hand, seems to emphasize the perlocutionary aspect of the metaphorical utterance (being prompted to a seeing-as) while maintaining that the locutionary act is unchanged whether a sentence is uttered metaphorically or literally. Bearing in mind that Austin explicitly acknowledges that he does not account for non-literal language use, Davidson’s theory of metaphor seems consistent with Austin insofar as it does not overcomplicate the locutionary act and emphasizes the non-conventional nature of metaphorical perlocution (Austin 119, 122).

¹⁷⁸Davidson, “What Metaphors Mean” 437.

¹⁷⁹Davidson, “What Metaphors Mean” 437. Similarly, Davidson later distinguishes lies from metaphors (“What Metaphors Mean” 442). Some elaboration on these points is desirable insofar as metaphoric language seems to be able to deceive and to be used in the service of some puns. This observation does not crucial enough to warrant specific examples.

¹⁸⁰Davidson 439.

is a sophisticated variation on this idea: the metaphorical meaning of a sentence is identical with the literal meaning of a corresponding simile, for some notion of “corresponding.”¹⁸¹ He rejects both of these views as incorrect based on the observation that similes are simple to understand insofar as “everything is *like* everything, and in endless ways,” whereas many metaphors are “very difficult to interpret and, so it is said, impossible to paraphrase.”¹⁸² It may be noteworthy that Davidson does not seem to immediately reject Nelson Goodman’s suggestion that “the difference between metaphor and simile is negligible.”¹⁸³

Arguably the biggest problem with “What Metaphor Means” is its failure to provide any discussion of what is meant by “literal meaning”; if the article is considered as a standalone piece, this is a grievous oversight. In particular, it seems to be the key factor behind two of Oliver Scholz’s criticisms of Davidson’s theory: he claims that it is unable to accommodate both metaphors relying on the false stereotypes that make up the “system of associated commonplaces” and metaphors containing fictional terms.¹⁸⁴ Related to this point is a weakness Davidson acknowledges: his theory is not equipped to deal with non-assertive speech acts such as questions and commands.¹⁸⁵ However, a significant proportion of Davidson’s other writings are dedicated to considerations of meaning that can potentially rectify these shortcomings. In “Truth and Meaning,” Davidson argues that the Tarskian definition of truth satisfies the sufficiency conditions for a theory of meaning. Davidson’s theory of meaning is a radical departure from earlier theories of meaning in that it consists only of Tarskian T-sentence theorems relating sentences in the object language to sentences in the metalanguage and therefore has no use for meanings per se, understood as ontologically distinct entities.¹⁸⁶ In particular, it is worth noting the drastic disparity between this conception of meaning and Grice’s notion of nonnatural meaning. Davidson briefly foreshadows his theory of metaphor in “Truth and Meaning”: “When we depart from idioms we can accommodate in a truth

¹⁸¹Davidson, “What Metaphors Mean” 439.

¹⁸²Davidson, “What Metaphors Mean” 440; emphasis mine.

¹⁸³Davidson, “What Metaphors Mean” 440.

¹⁸⁴Oliver Scholz, “‘What Metaphors Mean’ and how Metaphors Refer,” *Reflecting Davidson: Donald Davidson Responding to an International Forum of Philosophers*, Ed. Ralf Stoecker (New York: de Gruyter, 1993), 165.

¹⁸⁵Davidson, “What Metaphors Mean” 442.

¹⁸⁶Donald Davidson, “Truth and Meaning,” *Synthese* 17 (1967): 101–2. It is worth noting that Max Black’s discussion of meaning in chapter two of *Models and Metaphors* is quasi-Tarskian and also concludes that there are no entities called meanings (24).

definition, we lapse into (or create) language for which we have no coherent semantical account.”¹⁸⁷ It thus seems reasonable to assume that when Davidson says there is no such thing as metaphorical meaning, he is claiming that metaphorical interpretation cannot be reduced to necessary and sufficient truth conditions. In later writings, Davidson apparently recants some of his claims about meaning: “In my essay ‘What Metaphors Mean’... I was foolishly stubborn about the word meaning when all I cared about was the primacy of first meaning.”¹⁸⁸ By “first meaning,” Davidson means something roughly correspondent to literal meaning, the kind of meaning that can be found in dictionary entries; he implemented this jargon in hopes of distancing himself from various negative associations that “literal” has. In at least two articles (“A Nice Derangement of Epitaphs” and “Locating Literary Language”) Davidson uses the notion of first language to roughly sketch an account of implicated “meanings” that is similar to — but also markedly different from — that of Grice.¹⁸⁹ In particular, Davidson comments that Grice’s principles of implicature do not seem sufficient to handle all of the cases he wishes to address (notably, malapropisms and literature), and that they curiously lack a speaker intention factor. Interestingly, the nature and potential proliferation of malapropisms leads Davidson to ultimately conclude that “we should give up the attempt to illuminate how we communicate by appeal to conventions.”¹⁹⁰ This conclusion seems to suggest the existence of a rift between communication and formalization, an idea that is certainly worthy of further consideration. Davidson’s theory of meaning is intriguing and relevant, but also expansive and complex; exploring a portion of the contextual corona of “What Metaphors Mean” has been usefully elucidatory, but delving further into his account of meaning would be too tangential to the thrust of this chapter.

This chapter has presented a philosophical history of the concept of metaphor, from its ancient Greek origins where it referred to physical translocation and its Aristotelian meaning of a transfer of words, through centuries of disregard, slander, and hypocritical appeals for the

¹⁸⁷Davidson 106. This passage may be usefully relevant to considering the relationship between idiom and metaphor.

¹⁸⁸Donald Davidson, “Locating Literary Language,” *Truth, Language, and History* (New York: Oxford UP, 2005), 173.

¹⁸⁹Though he has acknowledged that one might legitimately use the word “meaning” for both first meaning and speaker meaning, Davidson continues to avoid the latter usage in his own writings.

¹⁹⁰Donald Davidson, “A Nice Derangement of Epitaphs,” *Truth, Language, and History* (New York: Oxford UP, 2005), 107.

total elimination of metaphor, and culminating in the twentieth-century renaissance in the philosophy of language and its myriad attempts to provide a positive, legitimating, rigorous account of metaphor by shifting the focus of inquiry from words to word-meanings, sentence-meanings, speaker-meanings, or beyond. The purpose of this undertaking was to provide a motivational and foundational context for the discussion of conceptual metaphor theory in chapter 3 and, in turn, support the discussion of the connexions between mathematics and metaphor in the remaining chapters. While there are hundreds more published accounts of metaphor that could be considered, the few discussed here constitute a sufficient sample insofar as they are among the most influential viewpoints and are jointly representative of the diversity in this field. None of the above theories seems obviously superior to the others, and the lack of consensus in the academic community is a testament to this. However, there are many insights into metaphor scattered throughout this chapter that range from the ubiquitous and nearly inane to the novel and nearly profound.

First, an obvious observation: metaphors do exist. That is, one encounters utterances or expressions or sentences in both spoken language and written text that are identified as metaphorical. Hence, any complete account of metaphor must be somehow connected to an understanding of language. However, the identification of metaphor is not as straightforward, obvious, and non-controversial a process as many theorists would have us believe. The application of the term “metaphor” seems somewhat obvious in a general sense, but becomes difficult when one begins to consider the details rigorously. Some metaphors are easily detected and identified, while others may only be recognized in retrospect. Once a metaphor is identified, the identifier may or may not be able to explain their reasoning for arriving at that conclusion. One widespread strategy, though it constitutes neither a necessary nor a sufficient condition, is the identification of falsity, absurdity, or some other violation of the conventions of language. While recognition of metaphor plays a crucial role in most traditional accounts of metaphor interpretation, some — notably Davidson’s — do not seem to require recognition for efficacy. Intimately connected to the issue of identification is the observation that linguistic meanings are context dependent: no sentence is definitively metaphorical (or non-metaphorical) apart from contextual considerations. These issues surrounding the identification of linguistic metaphors are one of the motivations for considering

conceptual theories in the next chapter: the intuition that mathematics involves metaphor conflicts with the fact that mathematical sentences such as “Odd numbers are not divisible by two” are rarely, if ever, identified as metaphorical.

There are several other points arising from the above discussion that a theory of metaphor should seek to account for. Every view of metaphor presents it as some kind of relation, between vehicle and tenor or figure and frame, for example. A fundamental issue is whether this relation is understood to be symmetrical or not. The current trend holds that metaphor is, in general, asymmetrical: “That surgeon is a butcher” and “That butcher is a surgeon” seem to express two very different thoughts, and “The sun is Juliet” seems nearly incomprehensible, for example. One’s answer to the symmetry question will have widespread ramifications for their theory, including implications for how metaphor may relate to other figures and tropes, such as simile. Another important point is that the idea of “dead metaphor” is ubiquitous in metaphor theories. This suggests a pervasive belief in a connexion between metaphor and linguistic development and evolution exists, a belief which merits attention. A final related point that theories of metaphor should be cognizant of is that metaphor itself is frequently understood by way of metaphors — what might be called metametaphors. Metaphors were originally conceived metaphorically as a “carrying across,” and contemporary authors continue to invoke metaphor in discussing metaphor (such as when they speak of dead metaphor, for example). Metametaphors may be taken to be abhorrent, acceptable as a useful communicative device, or ineliminably constitutive of our understanding of metaphor, but they should not be ignored. Symmetry of metaphor, metaphor death, and metametaphoricity are discussed as central issues in conceptual metaphor theory.

Despite the disparities between the views presented in this chapter and the disagreements they lead to, there is a unifying thread running through most of them. Whether semantic or pragmatic, descriptivist or referentialist, almost every view above subscribes to at least some of the core beliefs of the literal-truth paradigm. That is, each subsequent development in the metaphor theory in this chapter has been analogous to the addition of a corrective epicycle to a fundamentally flawed theory of celestial motion. As such, a certain set of objections applies pervasively:

[Most theories considered thus far] take falsity or anomaly as a criterion that

allows for the recognition of metaphor, and they all think of metaphorical interpretation as secondary, and based on processes that are rather different from those governing the interpretation of literal language. Moreover, all approaches [considered thus far] have problems with novel metaphors and the apparent ‘creation of similarity.’ In large part, these difficulties stem from the assumptions that literal language has an absolute priority over figurative language and that metaphor is essentially deviant, or at least distinct, from the literal.”¹⁹¹

Additionally, most theories of metaphor in the literal-truth paradigm fail to discuss the possibility of non-linguistic metaphors, including those occurring in other media (paintings, for example), and more radical alternatives such as Nicomachus’ cross-disciplinary application of skills (discussed near the beginning of this chapter).¹⁹² Such objections are largely responsible for the advent of conceptual metaphor and the paradigm shift it causes. Theories of conceptual metaphor grow out of the seeds planted by earlier skeptics of the literal-truth paradigm — including Vico, Nietzsche, and especially Richards — and are the subject of chapter 3. The development of the notion of conceptual metaphor is arguably the most important advent in metaphor theory from the perspective of someone interested in metaphor and mathematics. This is because the traditional view of mathematics as the ideal form of language use is as connected to the literal-truth paradigm as the traditional hostile views regarding metaphor are. In providing a serious alternative to the literal-truth paradigm, theorists simultaneously managed to empower and legitimate metaphor while making mathematics less untouchable, and thereby moved the two subjects close enough together for them to constructively associate for the first time.

¹⁹¹Leezenberg 135.

¹⁹²It should be noted that the absence of such explanations does not necessarily entail that none of the positions above can account for these phenomena.

CHAPTER 3

CONCEPTUAL METAPHOR

The locus of metaphor is not in language at all, but in the way
we conceptualize one mental domain in terms of another.¹

— George Lakoff

Chapter 2 explored some of the ways in which traditional theories of metaphor are problematic. The thesis common to most of these accounts is that metaphors are essentially linguistic phenomena and are necessarily derivative from established literal language. As I showed in chapter 2, this premise can lead to the problematic view that metaphors necessarily deviate from the truth and are unnecessarily obfuscating or merely ornamental, a sentiment that arguably reached its zenith with the logical positivist movement.² Perhaps more significantly, the fundamental place of “literality” in such theories means their proponents owe an account of this notoriously difficult notion. While a handful of scholars doggedly attempt to provide solutions to these problems from within their linguistic perspectives on metaphor, a new generation of metaphor researchers have instead adopted a radically different approach to metaphor.

A paradigm shift has been occurring in metaphor scholarship. A substantial number of contemporary metaphor scholars consider metaphor to be fundamentally a matter of thought rather than a primarily linguistic phenomenon. Although this approach has only gained popular support within the last few decades, it did not emerge *ex nihilo*. As suggested in chapter 2, notable progenitors of this hypothesis include Aristotle, Vico, Nietzsche, I.A. Richards, and Max Black. In particular, Richards might plausibly be identified as the father of this new movement based on his pronouncement that “[t]hought is metaphoric. . . and the metaphors of

¹George Lakoff, “The Contemporary Theory of Metaphor,” *Metaphor and Thought*, 2nd ed., Ed. Andrew Ortony (New York: Cambridge UP, 1993), 203.

²Skorupski 65–9.

language derive therefrom.”³ Many factors certainly contributed to the development of this new trend, but three are notably important to its emergence in the late 1970s as a serious alternative to the traditional view. First, none of the various linguistic theories of metaphors managed to prove themselves clearly superior to their competitors. Serious objections to each of the main linguistic accounts were then known and, indeed, persist to this day. Such a situation can provide motivation to investigate new or overlooked avenues. Second, the middle of the twentieth century saw the development of significantly improved empirical techniques and methodologies that allowed cognitive scientists to perform new research into how we think and reason. Many of these experiments generated results that were seen as incompatible with traditional understandings of language and cognition, and thereby provided impetus for the development of new theoretical frameworks. Third, though almost certainly related to the second point, there was a shift in attention within the philosophical literature on metaphor from the one-off poetically figurative sentences considered prototypically metaphoric by past scholars to the kind of ubiquitous, everyday metaphoricity that usually goes unnoticed; I discuss this distinction at greater length below. Though there are now multiple accounts of metaphor which hold it to be a basic part of cognition and not merely a fortuitous kind of linguistic failure, my attention in this chapter will be primarily focused upon the conceptual metaphor theory (CMT) proposed by George Lakoff and Mark Johnson.

I have opted to explore this particular cognitive theory of metaphor in detail for a number of reasons. Lakoff and Johnson were among the pioneers that ushered in this new paradigm in metaphor scholarship.⁴ Their seminal book *Metaphors We Live By* is an ambitious attempt to develop a novel theory of metaphor that overcomes the major problems associated with traditional linguistic views. Written in an accessible style, this work served as many people’s first significant exposure to the metaphor-as-thought movement. *Metaphors We Live By* is

³Richards 51; emphasis his.

⁴A few other scholars deserve mention here (besides the Newtonian giants mentioned in the previous paragraph). Lakoff and Johnson were significantly inspired by Michael Reddy’s work on the conduit metaphor in their development of the notion of conceptual metaphor (*Metaphors We Live By* 10). Harald Weinrich’s 1976 text *Sprache in Texten* posits that “verbal metaphors are not isolated occurrences but fall into semantically homogenic groups” (Müller 48–9) and that certain abstract notions simply cannot be grasped without using metaphors (Müller 50). Weinrich’s theory has strong parallels to Lakoff and Johnson’s work, but was developed independent of CMT and predates it by a few years; however, it has had minimal influence in anglophone scholarship as it has never been translated into English.

a provocative read that tends to elicit a strong reaction, and has accordingly been an important source of inspiration to many metaphor scholars, prompting support and criticism alike, and motivating the development of a variety of alternative theories. In the thirty years since this publication was released, Lakoff, Johnson, and a host of collaborators and advocates have generated a significant body of work on conceptual metaphor, its relationship to the embodied mind, and the implications of both. Conceptual metaphor theory is thus worthy of consideration as one of the most-established, best-known, and substantial — even if thoroughly controversial — theories of metaphor within the new cognitive paradigm. However, from the point of view of this dissertation, the principal reason for investigating CMT is that it forms the basis of the most important attempt to relate metaphor and mathematics yet published. *Where Mathematics Comes From*, co-written by Lakoff and Rafael Núñez, is a tour de force account of how the various branches of advanced abstract mathematics develop out of primitive cognitive capacities by means of conceptual metaphor. This chapter will be devoted to an examination of Lakovian CMT in general, while one of the foci of chapter 4 is to assess the account of mathematics grounded in this theory of metaphor.

It has been said that “[w]ithout concepts, there would be no thoughts.”⁵ If this plausible claim is accepted, then it follows that any theory of metaphor-as-thought must involve concepts, whether by relying on some specific account of concepts, having implications for how concepts are understood, or through some combination thereof. As is evident from its name, CMT is no exception: concepts are fundamental to CMT given its claim that our mental lives are extensively shaped by conceptual metaphors, cognitive mappings that allow structure to be constitutively imported into one conceptual domain from another.⁶ This posited deep connexion between metaphors and concepts is also the locus of many of the objections to the Lakovian approach. These tend to fall into two categories. The first type argues that the understanding of concepts entailed by the Lakovian notion of conceptual metaphor is unsatisfactory. The second type involves concerns about the relationship between concepts

⁵Jesse Prinz, *Furnishing the Mind: Concepts and their Perceptual Basis* (Cambridge: MIT Press, 2002), 1.

⁶Note that Lakoff et al. “use the term *cognitive* in the richest possible sense, to describe any mental operations and structures that are involved in language, meaning, perception, conceptual systems, and reason” (Lakoff and Johnson, *Philosophy in the Flesh* 12). I mention this to avoid confusion, as many philosophers adopt a narrower understanding of “cognitive.”

and language espoused by CMT. Given the centrality of concepts to CMT, I start by briefly introducing the notion of “concept,” thereby providing necessary context for understanding CMT. I follow this with a synopsis of CMT, starting with its roots in *Metaphors We Live By* and tracing key developments to the theory that have emerged over the last three decades. The final section of the chapter presents a defense of CMT against a variety of objections. My objective is to defend the general claim that metaphor is not a merely linguistic phenomenon, not to resolve the technical differences between apparently similar accounts of metaphor-as-thought. Inspired by the work of Cornelia Müller, I conclude by suggesting that in general, metaphor can not be reduced to either the conceptual or to the linguistic but is best understood as involving the dynamic interplay between them.

3.1 Theories of Concepts

“Concept” is a complicated and controversial term with a long and varied philosophical history; it is beyond the scope of my project to provide a comprehensive analysis of it.⁷ There are many competing philosophical accounts of concepts in the literature, none of which is decisively victorious. Rather than surveying the many accounts of concepts, I adopt a general perspective in order to bypass controversial details and focus on aspects of concepts there is some agreement about. This general discussion provides a common ground for considering Lakoff’s specific theory of concepts. Concepts are that which is essential to most human cognitive activity; for example, the concept BIRD is utilized when a person speaks about some bird, recognizes something as a bird, reasons about birds, etc. The word “concept” is used in everyday English to refer both to specific understandings of things held by individuals, as well as to collective understandings associated with groups of individuals, often without recognizing a distinction between these two.⁸ However, a more definite understanding of

⁷Readers whose interest is not sated by my brief discussion can find *many* works written by philosophers and cognitive scientists on the topic of concepts. A few noteworthy examples include Jerry Fodor’s *Concepts: Where Cognitive Science Went Wrong* (1998), Gregory Murphy’s *The Big Book of Concepts* (2002), Jesse Prinz’s *Furnishing the Mind: Concepts and Their Perceptual Basis* (2002), and the Eric Margolis and Stephen Laurence-edited volume *Concepts: Core Readings* (1999).

⁸In this chapter, I avoid using the word “concept” in a non-technical sense as much as possible to avoid confusion. I will make it clear, either explicitly or by way of context, if I am using the term to refer to some specific understanding of “concept.”

concepts than this is desirable in philosophy, where the notion plays important explanatory roles in theories of mind, language, and meaning. Philosophers have been theorizing about concepts almost as long as they have been using them, trying to clear up some of the ambiguity and imprecision found in the common understanding. Indeed, that the attempt to bring heightened clarity and rigor to a notion is referred to as conceptual analysis indicates how central the idea of “concept” is to much of philosophy.⁹ How conceptual analysis proceeds will depend upon which theory of concepts the philosopher holds.

Navigating the bewildering assortment of conceptual theories can be a daunting task. Thankfully, help is available. Jesse Prinz’s seven desiderata for a theory of concepts provide a useful framework for thinking about the relative merits of competing accounts; I will use these criteria to evaluate the classical theory of concepts and compare it to some leading alternatives.¹⁰ Two of Prinz’s desiderata are primarily concerned with requiring theories to accommodate empirical data. The first desideratum concerns the scope of a theory. Concepts are observed to come in a wide variety of flavours: they possess varying levels of abstraction; they may be theoretical, or formal, or natural; they may be fleeting or persistent; etc. Any theory of concepts must either be able to account for this observed variation in the kinds of concepts we are able to possess or to somehow explain it away.¹¹ The second desideratum acknowledges that a connexion exists between concepts and how we categorize the world and therefore an adequate theory of concepts ought to fit current empirical data on categorization. Cognitive scientists — notably Eleanor Rosch — have studied both category identification (identifying the category that an object belongs to) and category production (recognizing what attributes an object possesses given that it belongs to a certain category) producing data relevant to theories of concepts; I will discuss the specific observed effects below.¹²

Three of Prinz’s desiderata deal explicitly with the content of concepts. The intentional

⁹I am using “conceptual analysis” very broadly here to refer to the practice of attempting to improve our understanding of some concept through philosophical discussion and argumentation rather than the narrower, more technical usage found in early analytic philosophy.

¹⁰Other authors give conditions they believe a theory of concepts must satisfy (for example, chapter 2 in Fodor (1998) contains “five non-negotiable conditions on a theory of concepts” (23)). I have chosen to use Prinz’s because they seem suitably broad in scope (they subsume Fodor’s conditions, for example) and are relatively impartial in tone.

¹¹Prinz 3.

¹²Prinz 9–10

content desideratum states that a theory of concepts should account for how concepts come to “represent, stand in for, or refer to things other than themselves.”¹³ To meet this condition, a theory will probably have to say something about what these intentional contents are, whether they are understood to be natural kinds, ad hoc categories, or some other thing.¹⁴ Intentional content does not seem to exhaust conceptual content, as shown by various puzzles of reference, notably those that inspired Frege’s distinction between *Sinn* (sense) and *Bedeutung* (nominatum) as well as Hilary Putnam’s Twin Earth scenario. Prinz therefore includes a desideratum of cognitive content that states a theory should explain how concepts with identical intentional content can differ and how concepts with different intentional content can be alike.¹⁵ The final desideratum of content claims that the apparent unbounded productivity of our cognitive capacities has its root in the compositionality of our concepts. Compositionality is the idea that we possess many compound concepts and that the content and structure of such concepts functionally depends upon their constituent concepts. A finite number of combination rules for forming compound concepts together with a finite number of primitive concepts would theoretically allow for the creation of an infinite number of concepts.¹⁶ Prinz clarifies that this criterion requires that both intentional and cognitive content be compositional.¹⁷ The compositionality condition is central to the dispute between contemporary theories of concepts.

Prinz’s two final desiderata have to do with the active life of concepts. The acquisition desideratum posits that a theory of concepts should provide — or at least be compatible with — some credible account of how humans acquire concepts. This explanation must account for both the ontogeny and phylogeny of concepts if the two are distinguishable in the given theory. At the core of the acquisition condition lies the question of whether concepts are innate or learned.¹⁸ Intimately related to questions of concept acquisition is the thought that concepts must be public. The publicity desideratum says that both the intentional and cognitive content of a concept “must be capable of being shared by different individuals and

¹³Prinz 3.

¹⁴Prinz 4.

¹⁵Prinz 7–8.

¹⁶Prinz 12–3.

¹⁷Prinz 14.

¹⁸Prinz 8–9.

by one individual at different times.”¹⁹ This is a necessary condition if concepts are to play an explanatory role in linguistic communication and in intentional behaviour.²⁰

Prinz considers two further desiderata regarding the relation of concepts to language, but ultimately rejects them. The first is that “concepts simply are the meanings of words,” and Prinz abandons this because it rules out popular reference-based semantic theories.²¹ The second is that “public language is necessary for the possession of concepts,” a claim based in Wittgenstein’s private language argument that is controversial among most cognitive scientists and many philosophers.²² As Prinz wants to keep open the possibility that “one can present a theory of what concepts are without mentioning language,” he does not include any desiderata specifically about language in his list.²³ However, if CMT is going to use conceptual metaphors to account for the metaphors we observe in language then it must necessarily say something about the connexion between language and concepts. Keeping this proviso in mind, Prinz’s seven desiderata provide a framework that can be used to evaluate the classical theory of concepts, as well as subsequent views, including CMT.²⁴

From the time of Socrates until the last century, most scholars based their analyses of concepts in some version of what is now called the classical theory of concepts. The classical theory of concepts is, in fact, a “diverse family of theories centered around the idea that concepts have definitional structure.”²⁵ The idea that concepts are essentially equivalent to definitions — exhaustive lists of necessary and sufficient membership criteria — remains influential to this day. Because membership criteria for any given concept are given in terms of other concepts, a systematic hierarchy of concepts exists in the classical theory. One

¹⁹Prinz 14.

²⁰Prinz 14–5.

²¹Prinz 17.

²²Prinz 18.

²³Prinz 21.

²⁴Prinz’s desiderata are intended primarily as a framework for assessing whether a theory adequately accounts for conceptual structure, and provide only minor constraints on ontological concerns. I too am mostly interested in conceptual structure and am more or less indifferent to ontology, that is to whether concepts are mental representations or cognitive capacities, for example. That being said, I am leery of ontologies positing conceptual atomism (insofar as it is incompatible with conceptual metaphor) or concepts as abstract entities (as counterproductive to attempts to explain away abstract mathematical entities). For the purposes of this dissertation, I must therefore reject any theory of concepts that assumes or implies either of these positions.

²⁵Stephen Laurence and Eric Margolis, “Concepts and Cognitive Science,” *Concepts: Core Readings*, Eds. Eric Margolis and Stephen Laurence (Cambridge: MIT Press, 1999), 9–10.

can already see traces of the classical view in Plato's writings: in several dialogues, Socrates engages various characters in a collaborative conceptual analysis aimed at limning the essence of JUSTICE, PIETY, LOVE, etc. It is clear from the definitional nature of concepts in the classical view that they are intimately connected with categories; for example, defining a triangle as a polygon with exactly three sides gives membership criteria for the class of triangles. In Plato's version of the classical view, if human beings ever possess true knowledge then a deep connexion must exist between at least some mental concepts and the eternal Platonic Forms, as instantiated triangles in the world are related by virtue of being mere approximations of the same Form. The ancient Greeks and Romans combined Plato's philosophy with the theory of categorical logic developed in Aristotle's *Organon* and established the classical view of concepts that was dominant for two millennia. Later philosophers held variants of the classical view that differed from that espoused by the ancients but retained the central role of definition; for example, Locke's theory of ideas differs starkly from Plato's theory of Forms but the differences shroud a common core. It is a testament to the staying power of the classical theory that the contemporary English word "concept" has undergone relatively little change in meaning since its origin in the Latin term *conceptum*.²⁶

Classical theories do an excellent job of satisfying many of Prinz's criteria, which perhaps helps explain their long reign. Laurence and Margolis suggest that the classical theory offers "unified accounts of concept acquisition, categorization, epistemic justification, analytic entailment, and reference determination, all which flow directly from its basic commitments."²⁷ Prinz observes that the classical theory also appears to satisfy the cognitive content and publicity conditions.²⁸ The idea that concepts are definitions is a powerful one that does a lot of explanatory work both elegantly and economically.²⁹ Unfortunately, there are many

²⁶"concept, n." *OED Online*, Mar. 2014, Oxford UP, 25 Apr. 2014. The noun *conceptum* derives from the verb *concupere*, a word whose primary meaning was "to take inside," as in the conceiving of a child, but that also sometimes meant "to perceive by means of the senses," "to express in formal language, frame, draw up," and, most relevant to current considerations, "to conceive or grasp in the mind, form an idea of, imagine." All of these meanings were known in antiquity; the latter usage was available at least as early as the first century BCE, as it can be found in Cicero's writing ("concupiō", *Oxford Latin Dictionary*). The metaphor extending the idea of pregnancy from the physical to the mental sphere, allowing concepts and ideas to be understood as offspring of the mind, can be traced back at least as far as Plato's writings; it is seen in Socrates' tale of Diotima's teachings in the *Symposium*, for example (Plato, *Symposium*).

²⁷Laurence and Margolis 10.

²⁸Prinz 38.

²⁹For example, in the classical theory, the concept LITERAL is composed of necessary and sufficient conditions

reasons to think that the classical theory of concepts is untenable. One strong reason to reject the classical theory of concepts goes all the way back to Plato: it seems difficult, if not impossible, to come up with adequate definitions for most concepts; as Laurence and Margolis put it “there are few, if any, examples of definitions that are uncontroversial.”³⁰ While definitions for concepts like LOVE and JUSTICE are perennially controversial, even seemingly straightforward definitions such as “a bachelor is an unmarried man” can be seen as problematic under mild scrutiny (is the Pope a bachelor? A widower? A man in a committed monogamous common-law relationship? A single 15-year-old male? A single 25-year-old transsexual?). Wittgenstein’s discussion of family resemblance concepts in his *Philosophical Investigations* also provides relevant evidence against the classical view. Famously, Wittgenstein argued that the concept GAME does not admit of a necessary condition, that there is no essence that all games have in common. Rather, he believed that GAME and similar concepts are unified as a chain is, with each constituent part overlapping *some* others without overlapping *all* others.³¹ Such considerations show that the classical theory does not satisfy the scope requirement because “most of the concepts we use are impossible to define.”³² Another reason to be apprehensive about the classical theory comes from Quine’s “Two Dogmas of Empiricism.” The view that concepts are definitions seems incompatible with Quine’s conclusions about analyticity: “If there is no principled way to distinguish analytic beliefs from collateral knowledge, definitions devolve into unwieldy, holistic bundles. . . [t]hus, until definitionists provide a principled analytic/synthetic distinction, they have difficulty explaining how concepts can be shared.”³³ These philosophical objections revolve around the idea that many concepts possess an indeterminacy that seems fundamentally incompatible with their being definitions. Based on these objections and others, many philosophers have abandoned the classical theory of concepts.³⁴

and, therefore, my worries about the vagueness of the notion would seem to be unfounded!

³⁰Laurence and Margolis 15.

³¹Ludwig Wittgenstein, *Philosophical Investigations*, Trans. G.E.M Anscombe (Oxford: Basil Blackwell, 1972), 31^e–32^e.

³²Prinz 41.

³³Prinz 41. Scholars differ on the strength of this objection depending on their interpretation of Quine’s argument (Laurence and Margolis 20–1).

³⁴It should be noted that a rejection of the classical view of concepts typically does not necessarily entail abandonment of definitions in general. Coming up with good definitions for terms can be exceedingly fruitful even if we acknowledge that those definitions will rarely exhaust the concepts associated with those terms.

The community of cognitive scientists has been nearly unanimous in its rejection of the classical theory, due in large part to the experimental findings of Eleanor Rosch. If the classical theory is true, then the necessary and sufficient membership conditions comprising a concept should form a clear, well-defined partition between members and non-members of the associated category. However successful definitions may be at distinguishing members from non-members of a category, they do not seem equipped to make distinctions between any two members of their category; that is, a definition can only define the boundary of a category, it cannot provide any internal structure. In the early 1970s, Rosch and her colleagues performed a series of groundbreaking experiments demonstrating that many concepts possess an internal structure in the form of a typicality gradient, a fact that most psychologists have taken to be fatal to the classical theory. This research asked subjects to categorize a variety of exemplars, rating the typicality of each category member. A comparison of the reported typicality with the measured speed of answer showed that people do not take all members of a category to be on a par with each other, but rather take some members as more typical representatives of a class than others. For example, given the category BIRD, people tend to judge robins as very typical representatives, chickens and vultures as moderately typical, and ostriches and penguins as atypical.³⁵ In addition to showing that intracategorical structure is graded rather than homogeneous, Rosch found that intercategory structure exists: just as some members of a given category are more typical than others, some categories are more basic than others. Experiments show that, given an object, people are likely to categorize it at an intermediate level between the general and the specific. For example, people are typically faster to recognize an object as a dog than as a spaniel or an animal. Research also shows that these *basic-level concepts* seem to be acquired earlier in development.³⁶ Because the classical theory does not seem to be able to explain typicality effects or basic-level categorization, it fails to satisfy the categorization desideratum and, arguably, the acquisition criterion as well. Rosch's findings about categorization were thus an important source of motivation for Lakoff in his development of CMT.³⁷

The classical theory of concepts fails because it does not adequately satisfy at least the

³⁵Laurence and Margolis 24–5.

³⁶Prinz 10.

³⁷George Lakoff, *Women, Fire, and Dangerous Things* (Chicago: U of Chicago P, 1987), 15.

scope and categorization conditions. In the wake of Rosch’s research, several alternative theories have arisen and been put to good use, though no one of them has emerged as clearly superior. The first serious alternative approach was prototype theory, a cluster of viewpoints unified by the idea that “most concepts — including most lexical concepts — are complex representations whose structure encodes a statistical analysis of the properties their members tend to have.”³⁸ According to prototype theory, a concept is a mental representation of an idealized instance that possesses the maximal number of typical properties for some category.³⁹ Intentional content is defined by resemblance to the prototypical representation; thus, the possession of disjoint subsets of typical properties may be sufficient for category membership.⁴⁰ Prototype theory was strongly motivated by Wittgenstein’s discussion of family resemblance and Rosch’s research into typicality effects.⁴¹ It is no surprise, then, that one of its major strengths is being able to account for the typicality effects that help comprise the categorization desideratum. Concept members possessing a larger allotment of typical features tend to be identified as better representatives of that concept, and thus graded intra-categorical structure is explained by prototype theory. The abandonment of strict necessary and sufficient membership conditions makes prototype theory impervious to the main criticisms of classical theory rooted in definitional structure, yet it has at least two weaknesses of its own. First, many concepts seem to lack prototype structure and therefore prototype theory arguably fails to satisfy the scope desideratum. The most accessible examples may be complement concepts such as NOT-A-DOLPHIN (is a hamburger a more typical not-dolphin than a saxophone, a crow, or a plastic scale model of a dolphin?), but many other examples exist including empty or uninstantiated concepts such as ANCIENT ROMAN SPORTSCAR.⁴² Second, prototype theories seem to have difficulty meeting the compositionality condition as suggested by the following example: mercury is arguably *the* prototypical LIQUID METAL, yet it is a relatively atypical instance of METAL (compared to, say, iron) and of LIQUID (compared

³⁸Laurence and Margolis 27.

³⁹Prinz 52.

⁴⁰Prinz 75.

⁴¹Laurence and Margolis 28.

⁴²Laurence and Margolis 35–6. One possible line of defense is to deny that complement concepts based upon a negation are genuine concepts based on the artificiality of their construction. However, it does not seem likely that this defense will work for all conceivable examples.

to water).⁴³ While some scholars attempt to amend prototype theory to overcome these criticisms, others take them to be fatal and thus have sought other approaches.

A more recent alternative to the classical and prototype theories is the theory-theory of concepts.⁴⁴ This approach is built around an analogy between how concepts are interrelated and how the terms in a scientific theory are related; more specifically, the theory-theory holds that a concept “is constituted by its role in an explanatory schema.”⁴⁵ Concepts are thus “mini theories of the categories they represent.”⁴⁶ Theory-theory occupies an appealing middle ground between the rigid necessary and sufficient conditions of the classical theory and the loose correlational similarity condition of the prototype theory. Through its core analogy, theory-theory allows the voluminous literature on scientific theorization to inform our understanding of concepts. Its major advantages over prototype theory involve being better able to account for concepts with category membership based on more than mere resemblance, such as unobservable properties and causal connectedness.⁴⁷ Theory-theory emphasizes causal and explanatory relations among category features, and is thus better able to incorporate the observed human tendency towards essentialization in classification than classical and prototype theories are. Theory-theory also has the advantage of being able to import ideas about scientific theory development to account for conceptual development in humans over time.⁴⁸ However, theory-theory has some difficulties in accounting for the intentional content of concepts (“How do theories refer to categories?”), their cognitive compositionality (“[t]heories, in all their cumbersome complexity, are not the kinds of things that can be easily combined”), and their publicity (“[i]t is very unlikely that any two people have exactly the same theories of the categories they represent”).⁴⁹ While the theories discussed thus far have differed in the main only on the kind of conceptual structure they posit, others,

⁴³Laurence and Margolis 39. The compositionality criticism of prototype theory has frequently been championed by Fodor. Prinz, chapter 11, argues that prototypes exhibit a version of compositionality that is weaker than Fodor requires but is sufficient to satisfy the desideratum.

⁴⁴The theory-theory of concepts should not be confused with the theory-theory of mind. Gregory Murphy, a notable advocate of a version of the theory-theory of concepts, refers to his view as the “knowledge approach” to avoid this potential confusion (Murphy, *The Big Book of Concepts* 61).

⁴⁵Laurence and Margolis 45.

⁴⁶Prinz 76.

⁴⁷Prinz 77.

⁴⁸Laurence and Margolis 45–6.

⁴⁹Prinz 86–7.

such as Fodor’s atomism, take a totally different approach.

Like the theory-theory, conceptual atomism emerges in reaction to the failures of the prototype and classical theories, though the solution it proposes is more radical.⁵⁰ Conceptual atomism holds that lexical concepts have *no* structure whatsoever; they are primitives that do not decompose into properties or features but have a content “determined by the concept’s standing in an appropriate causal relation to things in the world.”⁵¹ The strengths of conceptual atomism include naturally satisfying the intentional content desideratum in a way that bypasses concerns regarding error that plague other theories. Conceptual atomism asserts that the causal relation between a concept and its instances allow for direct reference, thus avoiding complications caused by reference-mediating entities such as definitions, prototypes, and theories.⁵² Its rejection of most types of conceptual structure makes it immune to many of the most telling arguments against its rival positions, but makes it vulnerable to others.⁵³ Some argue that primitive concepts must be innate, which suggests that the conceptual atomist position must be radically nativist about concepts.⁵⁴ This means that conceptual atomism probably fails to satisfy the acquisition desideratum: postulating that the ancient Greeks possessed the concepts *CARBURETOR* and *CANADIAN*, for example, is not a plausible acquisition story.⁵⁵ Because of its emphasis on direct reference to worldly things, conceptual atomism also has difficulties with the cognitive content condition, notably with distinguishing between coextensive concepts, including those that are empty because they refer to nonexistent entities. That is, it seems that conceptual atomism not only has difficulty distinguishing *SUPERMAN* from *CLARK KENT*, but may even have trouble telling *SUPERMAN*

⁵⁰Fodor 93.

⁵¹Laurence and Margolis 60. A lexical concept is one that can be expressed by a single word, such as *BIRD* (Prinz 22). This qualification is important because conceptual compositionality requires that non-atomic compound concepts such as *YELLOW BIRD* also exist.

⁵²Prinz 90.

⁵³Laurence and Margolis 62.

⁵⁴Primitive concepts cannot be decomposed into more basic concepts. They are the bedrock that prevents compositional processes from falling into infinite regress (Prinz 94).

⁵⁵Laurence and Margolis 62–3. One might argue that *CARBURETOR* and *CANADIAN* are not primitive concepts (while this seems plausible, it does not seem immediately obvious exactly what primitives they are then composed of), and that logical atomism only requires primitive concepts to be innate. Fodor has defended his version of conceptual atomism against objections to nativism but there is no room or need to discuss this debate here.

and LEX LUTHOR (or, for that matter, SUPERMAN and UNICORN) apart.⁵⁶ In particular, this criticism seems to indicate an incompatibility between conceptual atomism and the belief that mathematical entities are non-existent or fictional. Furthermore, as we shall see shortly, CMT posits that even the most elementary concepts possess internal structure and is therefore also incompatible with conceptual atomism. Because of these incompatibilities with key elements of this dissertation, and the lack of a definitive argument in its favour, I shall discuss conceptual atomism no further.

What conclusions should be drawn from this discussion of concepts? First, empirical and philosophical developments over the last hundred years have led many scholars to reject the classical theory of concepts, principally because of its failure to satisfy the scope and categorization criteria. Thus, it would seem prudent to avoid importing this understanding of concepts into metaphor research, or into this dissertation. Note, however, that rejecting the idea that concepts are constitutively definitional does not mean concluding definitions have no place in philosophy or conceptual reasoning. Second, none of the leading approaches in contemporary concept research seem clearly superior to the others; each do an excellent job of satisfying some of Prinz’s desiderata while failing to satisfy others. It is possible that concepts may possess multiple kinds of structure and a single comprehensive theory of the type sought thus far might be impossible.⁵⁷ As we shall see, the theory of concepts underpinning CMT is not a clear instance of any one of the major approaches to concepts. This should not be regarded as sufficient grounds for suspicion toward this view; indeed, this rather seems to be a boon insofar as it means that CMT is not immediately subject to the standard objections leveled at the leading theories. Bearing this in mind, I will now move my attention to the details of CMT.

3.2 Conceptual Metaphor Theory

Metaphors We Live By begins with a claim that is the core premise of conceptual metaphor theory: “metaphor is typically viewed as characteristic of language alone, a matter of words

⁵⁶Laurence and Margolis 69.

⁵⁷Laurence and Margolis 72.

rather than thought or action. . . We have found, on the contrary, that metaphor is pervasive in everyday life, not just in language but in thought and action. Our ordinary conceptual system, in terms of which we both think and act, is fundamentally metaphorical in nature.”⁵⁸ An additional qualification clearly shows Lakoff and Johnson are directly opposed to at least some traditional ideas about metaphor. They tell us that “[m]etaphor is primarily a matter of thought and action and only derivatively a matter of language.”⁵⁹ Thus, while many agree with Hobbes that metaphorical language is an in-principle eliminable impediment to the clear communication of ideas, Lakoff and Johnson propose that metaphors in speech and writing are, rather, merely symptoms of metaphoricity inherent in the concepts underlying our language and its use. Developing this notion of conceptual metaphor and exploring its ramifications is, indeed, the primary objective of *Metaphors We Live By*; as such, almost every instance of the word “metaphor” in that text refers to conceptual metaphor and *not* to the narrower idea of metaphorical language.⁶⁰ This is noteworthy because many of the objections to CMT arguably depend on an equivocation that stems from misinterpreting Lakoff and Johnson’s claims about conceptual metaphor as pertaining directly to linguistic metaphors. Conceptual metaphor is intended to explain metaphorical language: the two are not identical. As Lakoff and Johnson say, “[m]etaphors as linguistic expressions are possible precisely because there are metaphors in a person’s conceptual system.”⁶¹

Lakoff and Johnson, however, do not offer a concise definition of conceptual metaphor at the beginning of *Metaphors We Live By*, but instead approach the idea through a series of examples intended to induce understanding. The clearest description of conceptual metaphor is given twenty years and thousands of pages later in *Where Mathematics Comes From*: a conceptual metaphor is **“a grounded, unidirectional, inference-preserving cross-domain mapping — a neural mechanism that allows us to use the inferential structure of one conceptual domain to reason about another.”**⁶² This dense definition surely begs

⁵⁸*Metaphors We Live By* 3. It is thus clear from the very first paragraph that CMT presumes a fairly specific, non-traditional notion of “concept”; I will however postpone explicit discussion of the way concepts are understood in CMT for the moment and focus first on the idea of conceptual metaphor.

⁵⁹Lakoff and Johnson, *Metaphors We Live By* 153.

⁶⁰Lakoff and Johnson, *Metaphors We Live By* 6: “whenever in this book we speak of metaphors. . . it should be understood that *metaphor* means *metaphorical concept*.”

⁶¹Lakoff and Johnson, *Metaphors We Live By* 6.

⁶²Lakoff and Núñez, *Where Mathematics Comes From* (New York: Basic Books, 2000), 6; emphasis mine.

to have its constituents unpacked.

Whether one understands **mapping** cartographically, mathematically, or more generally, it involves positing or representing a structural correspondence between two things. On a standard road map, for example, some features of the physical environment are represented by certain markings on a piece of paper. Describing metaphors as mappings seems quite apt, insofar as maps focus on certain features of the terrain while ignoring others entirely (for example, topographical maps and population density maps involve very different saliences). If a map became accurate in every detail and did not mask or omit any of the structure or features of what it represented, it would no longer be a map but rather a full-fledged identical copy of the thing represented. As with maps, it seems essential that metaphors involve partial rather than total mappings: identities are not metaphors. Although *Metaphors We Live By* makes no explicit use of mapping language in describing conceptual metaphor, it is consistent with and even gestures towards such a position in claiming that “[t]he essence of [conceptual] metaphor is understanding and experiencing one kind of thing in terms of another.”⁶³ The necessarily partial nature of metaphorical transference, on the other hand, is already explicitly present: “part of a metaphorical concept does not and cannot fit.”⁶⁴

Works subsequent to *Metaphors We Live By* in the CMT canon do explicitly discuss conceptual metaphor using mapping terminology. In “The Contemporary Theory of Metaphor,” for example, Lakoff says the following about the nature of conceptual metaphors qua mappings:

Mappings should not be thought of as processes, or as algorithms that mechanically take source domain inputs and produce target domain outputs. Each mapping should be seen instead as a fixed pattern of ontological correspondences across domains that may, or may not, be applied to a source domain knowledge structure or a source domain lexical item.⁶⁵

Describing conceptual metaphor in terms of static correspondences which may be activated rather than real-time algorithmic derivations coheres with the formal set-theoretic definition of a mapping as a collection of ordered pairs of elements and thus supports understanding

⁶³Lakoff and Johnson, *Metaphors We Live By* 5; emphasis theirs.

⁶⁴Lakoff and Johnson, *Metaphors We Live By* 13.

⁶⁵“The Contemporary Theory of Metaphor” 210.

conceptual metaphors specifically as *mathematical* mappings.⁶⁶ In more recent works, Lakoff and his collaborators have maintained that the claim “conceptual metaphors are mathematical mappings” is itself metaphorical since “mathematical mappings do not create target entities, while conceptual metaphors often do.”⁶⁷ Since 1997, however, they have focused their efforts on the development of a neural theory of language and have accordingly begun to theorize conceptual metaphors as neural mappings rather than mathematical mappings.⁶⁸ While there are reasons to appreciate their neural theory of conceptual metaphor, Lakoff and Johnson’s abandonment of the mathematical-mapping metaphor may have been somewhat hasty and unfortunate; I address this issue in chapter 5.

If conceptual metaphors are mappings then it seems obvious that they must be mappings *between concepts*. However, in the definition Lakoff uses the phrase “**conceptual domain**,” a phrase which he does not explicitly define. While it is possible that “conceptual domain” could be interpreted to simply mean “concept qua domain of a mapping,” I speculate that Lakoff’s use of this phrase is meant to convey that these mappings are not confined to isolated concepts but rather that they usually extend to include larger regions of connected and structured conceptual networks; it also reminds the reader that conceptual metaphors typically are partial in that they do not exhaustively map their entire input concept to their output concept, as noted above.⁶⁹ After their neurological shift, Lakoff and Johnson define the domains of metaphorical mappings as “highly structured neural ensembles in different regions of the brain.”⁷⁰

⁶⁶Saunders Mac Lane, *Mathematics: Form and Function* (New York: Springer-Verlag, 1986), 129. In most branches of mathematics, “mapping” is typically used as a synonym for “function.” A mathematical function is a relation between a collection of inputs (the *domain* of the function) and a collection of outputs (the *codomain*) such that each input is related to one output. However, Lakoff’s understanding of mapping is closer to a partial binary relation than a function as it does not seem to include the constraint that each input correspond to at most one output. This technical clarification is of little consequence at this point but becomes more pertinent in chapter 5.

⁶⁷Lakoff and Johnson, *Metaphors We Live By* 252.

⁶⁸Lakoff and Johnson, *Metaphors We Live By* 254.

⁶⁹It seems plausible that Lakoff may intend to invoke fellow cognitive linguist Ronald Langacker’s notion of a cognitive domain: “any sort of conceptualization: a perceptual experience, a concept, a conceptual complex, an elaborate knowledge system, and so forth” (Langacker, “An Introduction to Cognitive Grammar” 4).

⁷⁰Lakoff and Johnson, *Metaphors We Live By* 256. While Lakoff does ultimately wish to provide a neurobiological account of concepts, he does not seek to eliminate our phenomenological understanding; he describes himself as a “noneliminative physicalist.” As will be discussed below, CMT allows for multiple, legitimate, ineliminable levels of explanation; in my estimation, this is one of the theory’s major benefits (Lakoff and Johnson, *Philosophy in the Flesh* 108–14).

At the end of chapter 2 I argued that symmetry considerations feature among the properties of metaphor that any sufficient account of metaphor must address; this was motivated by examples of asymmetric linguistic metaphor such as the ubiquitous “That surgeon is a butcher.” In general, conceptual metaphors are **unidirectional**: understanding concept X in terms of concept Y is not the same as understanding concept Y in terms of concept X. Most conceptual metaphors are oriented in such a manner to allow us to comprehend a less well grasped concept in terms of a better understood concept. This is not to say that there are no instances where a distinct conceptual metaphor maps concept Y to concept X, or to rule out the possibility of genuinely bidirectional mappings between conceptual domains, metaphorical or otherwise. The claim is simply that the typical conceptual metaphor is unidirectional. This unidirectionality is related to one of conceptual metaphor’s other definitional properties, **cross-domainedness**. The power of metaphor seems to reside in its ability to connect disparate domains. Any mapping from some concept to itself does not involve seeing one domain in terms of another but only seeing some domain on its own terms. Unidirectionality and cross-domainedness are connected insofar as any mapping whose source domain and target domain are identical is bidirectional in a certain sense, though not necessarily invertible. It should be noted that, unlike metaphor, some kinds of conceptual mappings or relations — subcategorization and metonymy, for example — occur within a single domain.

Conceptual metaphor mappings do not involve random correspondences, but rather systematically preserve the overarching structure relating elements within a conceptual domain. By **preserving inferential structure**, conceptual metaphors enable a transfer of understanding that allows the comprehension of difficult, abstract, obscure, or alien concepts in terms of those better known; that is, conceptual metaphor “sanctions the use of source domain language and inference patterns for target domain concepts.”⁷¹ Thus, Lakoff and Johnson assert that “the preservation of inference is the most salient property of conceptual metaphors.”⁷² In more recent work, Lakoff and Johnson distinguish between *structural inferences* and *enacted inferences*, both of which can be preserved by conceptual metaphors. Structural inferences — such as inferring that Bollo is a mammal from the fact that Bollo

⁷¹Lakoff, “Contemporary Theory of Metaphor” 208.

⁷²Lakoff and Johnson, *Philosophy in the Flesh* 58.

is a gorilla — can arise non-imaginatively from the static structure of a conceptual system. Enacted inferences occur when reasoning about dynamical processes and involve an imaginative enactment of the process in question — if I tell you “Bollo beat his chest,” you may actively imagine a gorilla beating his chest and reach conclusions like “a thumping sound resulted” based on that imaginative project. Much of our reasoning involves an interaction of both kinds of inferences. It is important to note that in CMT enacted inferences are embodied — that is, are both allowed for and constrained by the particulars of our biological apparatus and physical experiences of a shared environment — and occur in the source domain. Thus, even though enacted inferences are dynamic, the correspondences making up conceptual metaphors need not be dynamic to preserve such inferences — just as fixed rods may transfer movement from a puppeteer to a puppet.⁷³

It is imperative to CMT that conceptual metaphors are not arbitrary mappings but are rather **grounded** in human embodiment and embodied experiences. Lakoff and Johnson have claimed that “most of our conceptual system is metaphorically structured; that is, most concepts are partially understood in terms of other concepts.”⁷⁴ If conceptual metaphors are arbitrary, this would entail that most concepts are arbitrary, a clearly problematic position unable to satisfy several of Prinz’s desiderata, notably publicity. In *Metaphors We Live By*, Lakoff and Johnson tell us that “conceptual metaphors are grounded in *correlations* within our experience”⁷⁵ but caution that they “do not know very much about the [actual] experiential bases of metaphors”⁷⁶ and thus their claims about the experiential foundations of specific conceptual metaphors are fairly speculative. These intuitions were later formulated into the Grounding Hypothesis. This hypothesis proposes that all concepts have at least some aspects that are semantically autonomous, in the sense that they emerge directly and non-metaphorically from regularities in our embodied experience, and that conceptual metaphors are grounded in this semantically autonomous structure.⁷⁷ While there might be some parallels between this hypothesis and the traditional idea that linguistic metaphors

⁷³Lakoff and Johnson, *Metaphors We Live By* 259–60.

⁷⁴*Metaphors We Live By* 56.

⁷⁵*Metaphors We Live By* 154–5; emphasis theirs.

⁷⁶Lakoff and Johnson, *Metaphors We Live By* 19.

⁷⁷Lakoff and Turner, *More Than Cool Reason* (Chicago: U of Chicago P, 1989), 113.

are grounded in underlying literality, Lakoff and Turner explicitly warn the reader that the Grounding Hypothesis is about concepts and not about language. They also remind us that it primarily concerns our body-dependent experiences rather than some human-independent reality and that it is consistent with the above claim that most concepts are not entirely semantically autonomous but rely on conceptual metaphor for much of their structure.⁷⁸

Citing experimental results in cognitive science, Lakoff and Johnson now hold that the Grounding Hypothesis has been confirmed. It is thus worth examining the details of their account of conceptual grounding. All embodied beings we know of rely on genetically determined discriminatory capacities for their survival. Even organisms such as plants and single-celled protists can be observed differentiating light from dark, up from down, food from non-food, threats from non-threats, etc. in their movements and reactions.⁷⁹ Thus, because many such discriminatory mechanisms are a hardwired part of human anatomy and physiology, we preconsciously and inescapably categorize the world in various fundamental and stable ways as we interact with it. Because many of our earliest and most basic experiences have to do with bodily survival in the physical world, they possess what CMT calls image-schematic structure, that is, a sparse sensorimotor form relating to our possible movements and actions. Such image schemata themselves possess an internal logic that allows for our action-guiding reasoning about the physical world.⁸⁰ Thus the particularities of human embodiment ensure that our experiences necessarily involve specific kinds of categorization, and non-metaphorical primitive concepts emerge from this foundation. There

⁷⁸Lakoff and Turner 119.

⁷⁹Lakoff and Johnson, *Philosophy in the Flesh* 17.

⁸⁰There is not enough room to present a thorough account of the research on image schemata here; more details can be found in almost any work by Lakoff and/or Johnson published since 1986. However, three key quotes will bolster the above synopsis:

- ◊ “An image schema is a recurring, dynamic pattern of our perceptual interactions and motor programs that gives coherence and structure to our experience” (Johnson, “Body in the Mind” *xiv*).
- ◊ “Image schemata are gestalt structures, consisting of parts standing in relations and organized into unified wholes, by means of which our experience manifests discernible order” (Johnson, “Body in the Mind” *xix*).
- ◊ “*image schemata are not propositional*, in that they are not abstract subject-predicate structures...that specify truth conditions or other conditions of satisfaction...They exist, rather, in a continuous, analog fashion in our understanding...On the other hand, *image schemata are not rich, concrete images or mental pictures, either*” (Johnson, “Body in the Mind” 23).

is extensive agreement between individuals' versions of these primitive concepts due to the universality of many of our basic embodied experiences, thanks to humans having developed in almost-identical environments according to almost-identical genetic instructions.⁸¹ Thus, this theory provides an explanation of the universality of primitive concepts without requiring innateness.⁸² Conceptual metaphors that have non-metaphorical spatiomotor concepts as their domains are said to be *primary metaphors* and are directly grounded; other *complex metaphors* obtain their grounding indirectly through their associations with such primary metaphors.⁸³ Much of Lakoff's recent CMT research is aimed at enriching this picture of primitive concept development with further neurological detail.⁸⁴

A substantial portion of the CMT corpus is devoted to discussing the specific conceptual metaphors that Lakoff and his coauthors believe constitute the core of our conceptual system and, although the above definition does a good job of introducing the idea of conceptual metaphor, some specific examples are useful for illustration. The input and output of a conceptual metaphor are referred to respectively as the *source domain* and *target domain*.⁸⁵ CMT denotes conceptual metaphors as follows: TARGET-DOMAIN IS SOURCE-DOMAIN.⁸⁶ Given the

⁸¹It may already be obvious to some readers that there are deep parallels between certain passages of Wittgenstein's writing and parts of CMT. For example, Wittgenstein's notion of "seeing-as," as discussed in *Philosophical Investigations IIxi*, seems related if not outright identical to the "seeing-in-terms-of" that is a core aspect of conceptual metaphor: "*The essence of metaphor is understanding and experiencing one kind of thing in terms of another*" (Lakoff and Johnson, *Metaphors We Live By* 5; emphasis theirs). At times, Wittgenstein seems wholly sympathetic to the cognitive linguistics project: "If the formation of concepts can be explained by facts of nature, should we not be interested, not in grammar, but rather in that in nature which is the basis of grammar?" (*Philosophical Investigations* 230^e). More specifically, the Wittgensteinian idea that our language games depend upon our specific form of life may be directly analogous to the Lakoffian notion that language is grounded in embodiment: "[human beings] agree in the *language* they use. That is not agreement in opinions but in form of life" (Wittgenstein, *Philosophical Investigations* 88^e). The idea that a shared human form of life provides the intersubjectivity necessary for linguistic communication also seems to be reflected in Wittgenstein's somewhat cryptic statement "If a lion could talk, we could not understand him" (*Philosophical Investigations* 223^e). Lakoff and Johnson do briefly mention Wittgensteinian forms of life, but only hint at the connexions between their work and his, mostly suggesting that he remained fettered by the assumptions of analytic philosophy despite his positive suggestions (*Philosophy in the Flesh* 450). Discussing the relationships between Wittgenstein and Lakoff in detail is beyond the scope of this dissertation.

⁸²Lakoff and Johnson, *Philosophy in the Flesh* 57. "Primitive" does not mean free from internal structure, but only that such concepts have a kind of priority.

⁸³This distinction will be discussed further below, with explicit examples provided. The interested reader is directed to pages 50 to 54 of *Philosophy in the Flesh*, where a list of representative examples of primary metaphor is provided.

⁸⁴See Gallese and Lakoff (2005) and Lakoff (2008) for more details.

⁸⁵One must be careful not to confuse these instances of the word "domain" with the technical mathematical usage described in an earlier footnote.

⁸⁶In more recent writings, Lakoff and Johnson sometimes use an alternative notation that will seem more

prevalence of traditional linguistic theories of metaphor, it is easy to forget that TARGET IS SOURCE is merely a notational mnemonic and not the metaphor itself; to avoid equivocation, it is imperative to remember that a conceptual metaphor is the collection of correspondences between portions of the source domain and the target domain.⁸⁷ I now turn my attention to several central examples of conceptual metaphor used by CMT.

The conceptual metaphor AFFECTION IS WARMTH is integral to how we experience and reason about affection.⁸⁸ This is evidenced, in part, by the unconscious ease with which we produce and interpret language involving this metaphor:

- ◇ We shared a warm embrace.
- ◇ Edith was hot for Darrell. Darrell, too, was burning with passion.
- ◇ My advances generated only a lukewarm response.
- ◇ She acknowledged him frostily.
- ◇ Olaf didn't stay the night because he is frigid.
- ◇ Sally is a block of ice.⁸⁹

According to CMT, these sentences are all metaphorical expressions of the same underlying conceptual metaphor; that is, producing or understanding these sentences involves conceptualizing affection in terms of warmth.⁹⁰ Moreover, it should be noted that all of these examples are more or less common everyday utterances, not ostentatious poetry: outside of the context of being presented as examples of metaphorical utterances, their metaphoricity likely would pass unnoticed. Speaking about affection in terms of warmth is normal and conventional, not exceptional. Whereas such cases force linguistic theories of metaphor to provide an account of “dead metaphors,” they cause no such complication for CMT because

natural and familiar to those with mathematical training: Source-Domain→Target-Domain. Despite these benefits, I will use the small capital letter notation because it is standard in the literature.

⁸⁷Lakoff, “Contemporary Theory of Metaphor” 207.

⁸⁸Lakoff and Johnson, *Metaphors We Live By* 255.

⁸⁹The last example is borrowed from Searle (“Metaphor” 82).

⁹⁰Of course, this charitably assumes that these sentences occur in a typical context, that is, one that connects them to this metaphor. One should remain cognizant of the contextuality of language throughout this discussion of conceptual metaphors.

understanding a sentence using concepts that are structured by conceptual metaphors does not require conscious recognition that this is taking place.⁹¹

AFFECTION IS WARMTH is a paradigmatic example of a primary conceptual metaphor. Primary metaphors “are directly grounded in the everyday experience that links our sensory-motor experience to the domain of our subjective judgments,”⁹² and, as such, are a foundational element of our conceptual system. Indeed, the universality of the collection of primary metaphors among humans is what allows me to speak about *our* conceptual system rather than *my* conceptual system. Yet primary metaphors are learned rather than innate: “[w]e acquire a large system of primary metaphors automatically and unconsciously simply by functioning in the most ordinary of ways in the everyday world from our earliest years.”⁹³ Lakoff and Johnson speculate that the AFFECTION IS WARMTH metaphor is acquired in the following way. When a parent is affectionate towards an infant, they typically snuggle close to them, thereby warming the baby with their body heat. These early simultaneous experiences of warmth and affection lead the infant to strongly associate the two domains.⁹⁴ In the neural version of CMT, the simultaneity of early experiences of warmth and affection are claimed to incite a *neural conflation*, a neural linkage between the temperature-devoted and emotion-devoted regions of the brain.⁹⁵ Pithily, Lakoff and Johnson tell us that this conflation arises because “neurons that fire together wire together.”⁹⁶ When affection and warmth are later

⁹¹Conceptual metaphor theorists hold that such conventional *entrenched* conceptual metaphors are thoroughly alive, as they enjoy copious (if unconscious) usage. A dead conceptual metaphor is a cross-domain mapping that is no longer in common usage. Lakoff’s canonical example involves the English word “pedigree,” which derives from *pie de grue*, Old French for “crane’s foot.” Whereas the metaphor connecting the branching of a family tree with the shape of a bird’s foot was once part of the concept of pedigree, this mapping has become etymological trivia rather than an active part of our conceptual system (Lakoff, “Death” 144).

⁹²Lakoff and Johnson, *Metaphors We Live By* 255.

⁹³Lakoff and Johnson, *Philosophy in the Flesh* 47.

⁹⁴While this account of the origins of the AFFECTION IS WARMTH metaphor seems plausible, it may be overly speculative and simplistic, a frequent criticism leveled against CMT. For example, Lakoff and Johnson do not discuss the possibility that hormones responsible for our experienced feelings of affection also play a role in body temperature regulation. There is some experimental evidence that pleasant one-on-one social interactions between adults are correlated with an increase in core body temperature (Dabbs and Moorer 518). Ultimately, such criticisms target explanations of how specific conceptual metaphors arise and would require substantial bolstering to upgrade them to a serious attack on CMT generally.

⁹⁵Lakoff and Johnson, *Philosophy in the Flesh* 48. Lakoff and Johnson credit Christopher Johnson — not to be confused with CMT pioneer Mark Johnson — with developing the theory of neural conflation they adopt. See C. Johnson (1999) for more details.

⁹⁶Lakoff and Johnson, *Metaphors We Live By* 256.

experienced separately, the associations persist, just as a channel remains even when a river is not actively flowing through it. In this way, hundreds of primary metaphors form automatically and unconsciously in all people under normal developmental circumstances, based upon the shared aspects of human embodiment; for example, AFFECTION IS WARMTH specifically depends strongly on our mammalian biology: we are warmth-radiating, temperature-sensitive beings that attend closely and lovingly to our young.

AFFECTION IS WARMTH is unidirectional, allowing us to understand affection in terms of warmth but not vice versa; for example, nobody would understand that you wanted to turn up the thermostat if you walked into a room and asked “Can we make the room a bit more loving?” Primary metaphors are naturally oriented from the sensorimotor to the subjective; this is because sensorimotor concepts possess more native inferential structure, often in the form of image schemata, than concepts based in abstract subjective experiences such as emotions.⁹⁷ For example, the temperature receptors in my skin constantly provide me with information about the warmth of my environment that allows me to take action to maintain my optimal body temperature. I experience warmth as possessing degrees of intensity as well as a directionality (that is, warmer than me or colder than me). Because these receptors are spread across my surface, different parts of my body may experience different intensities and thus I often also experience warmth as emanating from a source. These structural elements in the resulting concept of WARMTH allow me to make inferences like “If I wish to stop being cold, I should move away from the source of the coldness.”⁹⁸ When I think about affection, I metaphorically import inferential structure from my understanding of warmth: “If I don’t like that she is cold to me, I should distance myself from her.” We do not always understand affection in terms of warmth; for example, this metaphor may remain inactive while another metaphor — such as AFFECTION IS SWEETNESS — provides inferential structure.⁹⁹ Our concept of affection is largely structured by conceptual metaphors; without them, it only possesses

⁹⁷Lakoff and Johnson, *Philosophy in the Flesh* 55. This portion of Lakoff and Johnson’s account synthesizes Joseph Grady’s theory of primary metaphor and Srinivas Narayanan’s neural theory of metaphor. See Grady (1997) and Narayanan (1997) for more details.

⁹⁸While we may have some conceptual metaphors with warmth as the target domain, the important point is that the concept of warmth possesses a significant amount of inherent inferential structure, enough to directly ground any conceptual metaphor that features it as the source domain.

⁹⁹Lakoff and Johnson, *Philosophy in the Flesh* 56.

a minimal skeleton of structure. While we might theoretically be able to perform a small amount of reasoning based solely upon this literal skeleton of AFFECTION, it would take a considerable, and likely counterproductive, effort to isolate ourselves from the conceptual metaphors that also make a contribution.¹⁰⁰ While primary metaphors — such as AFFECTION IS WARMTH, MORE IS UP, SIMILARITY IS CLOSENESS, CATEGORIES ARE CONTAINERS, etc. — form the foundational ground of our conceptual system, a second large class of conceptual metaphors are not directly grounded.

Many of our most important conceptual metaphors have non-sensorimotor source domains with significant inferential structure. Lakoff and Johnson call these complex metaphors. Some examples include TIME IS MONEY, PEOPLE ARE PLANTS, A LIFETIME IS A DAY, and LOVE IS A JOURNEY. TIME IS MONEY is a paragon instance of a complex metaphor, one in which an abstract concept metaphorically lends its structure to another abstract concept: money is a complicated human creation, not a basic sensorimotor percept. In our culture, we make thorough use of this metaphor: we are constantly trying to think of new ways to save time, of worthwhile ways to invest our time rather than spending it frivolously, and so forth. We have institutionalized this metaphor in the practice of paying people's wages based on amount of time spent working, for example. However, other cultures — such as that of Pueblo communities — do not seem to conceptualize time as a resource. The absence of this metaphor from such cultures may be apparent in various ways, from the lack of linguistic expressions employing monetary language to describe temporal phenomena to a pace of life drastically different from ours thanks to the absence of time budgeting.¹⁰¹ Another example of a cultural variation in complex metaphor can be seen in one of the ways the future is conceptualized. It is common for humans to conceptualize themselves as moving observers going into a fixed future. However, in some cultures — such as my own — we tend to think of ourselves as facing the future and moving into it, while other cultures — such as the Aymara of the Andes — see themselves as facing the past and carefully walking backwards into an unseen future.¹⁰² Thus, while most primary metaphors are universal among humans, there seems to be room for variation in complex metaphors from culture to culture.

¹⁰⁰Lakoff and Johnson, *Philosophy in the Flesh* 58–9.

¹⁰¹Lakoff and Johnson, *Philosophy in the Flesh* 164–5.

¹⁰²Lakoff and Johnson, *Philosophy in the Flesh* 141.

Cultural variations in complex metaphors raise the question of the origins and grounding of metaphor. Lakoff and Johnson explain that complex metaphors are “built out of primary metaphors plus forms of commonplace knowledge: cultural models, folk theories, or simply knowledge or beliefs that are widely accepted in a culture.”¹⁰³ Thus, though complex metaphors may not have direct experiential grounding of their own they are indirectly grounded by way of their component primary metaphors.¹⁰⁴ For example, there is no simple experiential correlation between purposeful lives and journeys that could ground the complex metaphor A PURPOSEFUL LIFE IS A JOURNEY. Rather, Lakoff and Johnson claim this metaphor is grounded because it is the synthesis of two primary metaphors — PURPOSES ARE DESTINATIONS and ACTIONS ARE MOTIONS — combined with the cultural belief that every person has a purpose in life that they should strive to achieve.¹⁰⁵ Syntheses of primary metaphors like this are conjectured to arise by means of conceptual blending, a notion extensively developed by Gilles Fauconnier and Mark Turner.¹⁰⁶ A conceptual blend is an operation on two input mental spaces that yields a third blended space that “*inherits partial structure* from the input spaces and *has emergent structure* of its own.”¹⁰⁷ The blended space inherits structure from partial cross-space mappings between counterpart facets of the input spaces;¹⁰⁸ the blend also has emergent structure that arises through the related processes of composition (new relations forming between elements projected from the input spaces), completion (importing elements from cultural or other cognitive models and schemata to integrate disparate projected elements into coherent wholes), and elaboration (making conceptual inferences using

¹⁰³ *Philosophy in the Flesh* 60.

¹⁰⁴ Lakoff and Johnson, *Philosophy in the Flesh* 63.

¹⁰⁵ *Philosophy in the Flesh* 61.

¹⁰⁶ Conceptual blending theory is an alternative cognitive approach to conceptual structure that shares many motivations and premises with CMT. While Lakoff suggests that the two theories are “different in scope and intent” (Lakoff and Johnson, *Metaphors We Live By* 261), both he and others seem to believe that “the two approaches are complimentary” (Grady, Oakley, and Coulson 420). For more details on conceptual blending, see Fauconnier and Turner (2002).

¹⁰⁷ Fauconnier 149; emphasis his. Mental spaces are to conceptual blending theory as conceptual domains are to CMT, the objects that form the domains of the mappings under consideration. Some authors have suggested that mental spaces are partial, temporary representational structures that depend upon the more stable and systematic conceptual domains (Grady, Oakley, and Coulson 421). Considering the detailed differences between these two kinds of mental constructs is a task for another time.

¹⁰⁸ In some cases, these partial cross-space mappings are themselves conceptual metaphors (Grady, Oakley, and Coulson 428).

the emergent logic of the blend).¹⁰⁹ Lakoff and Johnson do not discuss the technical details of metaphorical blending, and neither will I. The important point here is that the myriad complex metaphors that structure our conceptual systems and cultural institutions are not problematically arbitrary but are grounded indirectly through associated primary metaphors and the semantically autonomous aspects of their domains; moreover, plausible suggestions have been made regarding mechanisms of complex metaphor formation.

If many primary metaphors are universal among humans, and complex metaphors are syntheses of primary metaphors, how can we explain the cultural variation observed in the structuring of fundamental concepts? The answer is that although there are extensive universal regularities in the embodied experience of humans, peoples of different cultures also have differing experiences based in the peculiarities of their local environment and their accidental history.¹¹⁰ Cultural beliefs and biases can enter into complex metaphors as the “common-place knowledge” component mentioned above. Conceptual metaphors, whether primary or complex, are grounded in experiential bases. It does not follow from this that having an experiential basis necessarily entails possession of an associated conceptual metaphor. As Lakoff puts it, “[e]xperiential bases motivate metaphors, they do not predict them. Thus, not every language has a MORE IS UP metaphor, though all human beings experience a correspondence between MORE and UP. What this experiential basis does predict is that no language will have the opposite metaphor LESS IS UP.”¹¹¹ Exactly what causes a conceptual metaphor to coalesce out of an experiential basis in some cases but not others is not explained in the Lakovian corpus.

Conceptual metaphors, whether primary or complex, emerge from and are grounded in

¹⁰⁹Fauconnier 149–51. Note that the process of completion posited by the conceptual blending theorists seems directly analogous to — if not outright identical to — how we import or create details to fill in the gaps between the relatively sparse descriptions presented in novels and other fictions.

¹¹⁰Lakoff has been criticized for not adequately defining key terms in his theory, including “culture.” I will discuss this criticism later in this chapter. I understand cultures in a fairly broad sense, as groups of people united through *shared* experiences, beliefs, values, etc.; thus, I am a member of many different cultures, of various cardinalities, some as small as 2.

¹¹¹“Contemporary Theory of Metaphor” 241. Note that this is a probabilistically motivated, empirically testable prediction about the conceptual metaphors grounding the various languages spoken by humans. It is not *impossible* that humans could possess the metaphor LESS IS UP, merely *unlikely*. I leave it up to cultural linguists and anthropologists to test this prediction. It may be worth noting that the quantity being tracked seems to make some difference here: a person who lacks sensibility is thought of as having their head in the clouds, for example. It would be interesting to hear what Lakoff would say about such examples.

experiential bases. Conceptual metaphors are also instantiated or realized in the world by humans. The most obvious realizations are linguistic, sentences that depend upon underlying conceptual metaphors for their construction and interpretation. In addition, conceptual metaphors are instantiated throughout the spectrum of human creations: in artistic works, in instruments and tools, in buildings, in social practices and observances, and in other cultural artifacts.¹¹² For example, the architecture of cathedrals relies upon metaphors like GOOD IS UP, with lofty cavernous arched ceilings being conceived as better tributes to the divine than squat huts. Historical timelines depend upon the metaphor TEMPORAL EXTENTION IS SPATIAL EXTENTION. Cartoons often invoke conceptual metaphors like ANGER IS HEAT visually: angry characters routinely transform into whistling kettles or soaring thermometers.¹¹³ Importantly, Lakoff notes that “[e]xperiential bases and realizations of metaphors are two sides of the same coin: they are both correlations in real experience that have the same structure as the correlations in metaphors. The difference is that experiential bases precede, ground, and make sense of conventional metaphorical mappings, whereas realizations follow, and are made sense of, via the conventional metaphors. And, as we noted above, one generation’s realizations of a metaphor can become part of the next generation’s experiential basis for that metaphor.”¹¹⁴ Thus, conceptual metaphors can become dynamically entrenched, originating in experiences of nature alone but later fortified by human-created instantiations of the metaphor within the shared environment. An analogous example is pertinent to the next chapter. There are few rectangles, circles, and regular polygons in natural environments untouched by human manipulation; the faces of certain crystals, the hexagonal cells of a honeycomb, and the circular visages of the moon and sun are among the short list of examples. However, some early humans gleaned these shapes from sparse natural sources such as those mentioned above, and human artifacts now frequently involve such shapes, such as the rectangular page you are currently reading. Indeed, rectangles and circles are now ubiquitous to the point of our experience being permeated with them, so much so that these shapes are among the first things children are formally educated about. It is one of the wonders of animal mimicry that

¹¹²Thus, the conceptual metaphors possessed by some culture, institution, or discipline are often constitutive of its identity.

¹¹³Lakoff, “Contemporary Theory of Metaphor” 241.

¹¹⁴“Contemporary Theory of Metaphor” 244.

we humans are able to reproduce and thereby amplify various elements of our experience, including conceptual metaphors.

A third kind of conceptual metaphor is discussed in several works in the CMT canon, but is conspicuously absent from *Philosophy in the Flesh* and other recent writings. In *More Than Cool Reason*, Lakoff and Mark Turner suggest that some metaphors involve correspondences between relatively specific mental images rather than between conceptual domains.¹¹⁵ Unlike other conceptual metaphors which underlie multiple expressions in language and culture, image metaphors are one-shot mappings because the “proliferation of detail in the images limits image mappings to highly specific cases.”¹¹⁶ For example, Kendall Walton’s example “Crotone is on the arch of the Italian boot” seems to be underpinned by an image metaphor that maps a mental image of a boot onto a mental image of the shape of Italy.¹¹⁷ This metaphor allows me to infer from the above sentence that Crotone is in the southeast of Italy. Due to their one-shot nature, image metaphors do not tend to play a deep structuring role in our conceptual system and are often merely aesthetically pleasing or communicative shortcuts rather than fundamental ways of thinking; for example, the sentences “Crotone is in southeastern Italy, on the northeast coast of the Calabrian peninsula” and “Crotone is located at 39° 05′ N 17° 07′ E” also say where Crotone is located without employing the superficial boot metaphor. Lakoff and Johnson have always been primarily interested in developing a comprehensive theory of embodied reason and image metaphors are only peripheral to this project; they seem to not be metaphors we live by but rather idiosyncratic metaphors that “stand alone and are not used systematically in our language or thought.”¹¹⁸ It is clear that many mathematical image metaphors are among the class of *extraneous* metaphors mentioned briefly in *Where Mathematics Comes From*, metaphors that “can be eliminated without any substantive change in the conceptual structure of mathematics.”¹¹⁹ Canonical examples of extraneous mathematical metaphors include functions that are named after the shape of

¹¹⁵Lakoff and Turner 89. Note that the “images” in question here may come from any experiential modality; they need not be visual in nature, though in practice they often are.

¹¹⁶Lakoff, “Contemporary Theory of Metaphor” 230.

¹¹⁷Kendall Walton, “Metaphor and Prop Oriented Make-Believe,” *European Journal of Philosophy* 1.1 (1993): 40.

¹¹⁸*Metaphors We Live By* 54.

¹¹⁹Lakoff and Núñez, *Where Mathematics Comes From* 53.

their graph, such as the step function (so called because its graph resembles a staircase) and cardioid (which vaguely resembles a heart). These descriptive names are based on superficial structural resemblances and are in no way constitutive of the mathematical objects in question or how we reason about them. Image metaphors are an important part of CMT qua theory of metaphor and may be progenitors of more persistent conceptual metaphors, but they do not play a significant role in structuring our conceptual system and are thus absent from many works of the CMT corpus.

The majority of the metaphors considered by CMT are conventional conceptual metaphors that are deeply entrenched in the way humans think. Such metaphors are frequently realized in our speech and writing, and typically are invoked without conscious recognition of their metaphoricity. Most earlier theories at most acknowledge these utterances as “dead metaphors” and give them little mention, focusing instead on more ostentatiously creative metaphors, whereas one of the fundamental insights of CMT is that it is reality as we experience it — and not just poetry — that is, in an important sense, metaphorical. Though this powerful insight regarding conventionalized “metaphors we live by” is its primary focus, CMT also delivers an account of novel metaphor. This is an important inclusion in the theory for at least two reasons. First, if CMT is to be seen as a legitimate alternative to existing philosophical theories of metaphor rather than a parallel theory that misses the point, it must say something about the phenomena that most other theories take as paradigmatic. Second, it would seem that even the most conventional conceptual metaphor must have once been new insofar as CMT holds conceptual metaphors to be acquired rather than innate. Thus, there is room for at least two kinds of novelty in the Lakovian account of metaphor: linguistic novelty and conceptual novelty.

First, consider linguistic novelty. Existing conventional metaphors may give rise to genuinely novel linguistic utterances. Through careful, clever construction, a master wordsmith may breathe new vitality into any idea, even one arising from the most mundane and ubiquitous conceptual metaphor. Such novel realizations may well involve extending the conceptual mapping to other elements of the source domain in a natural yet heretofore unexploited way. Consider the following example: the conceptual metaphor `PEOPLE ARE MACHINES` has been realized for many years in sentences such as “At the end of a ten-hour shift, I’m running on

fumes” and “I feel like I’m about a quart low.” A novel metaphorical sentence that realizes unexploited aspects of the MACHINES concept is “The kids these days seem to be running a new operating system that is incompatible with my aging hardware.” Novel extensions often reflect conceptual changes in the source domain arising from influences such as the introduction of new technologies; for example, the sentences above reflect the transition from an age where machines were exclusively mechanical to one where machines have a significant number of sophisticated electronic components. Under CMT, novel figurative utterances of this type cannot count as cases of novel *conceptual* metaphor because the underlying conceptual mappings have been previously established and conventionalized; the best they can do is blow some dust off of well-worn conceptual metaphors and draw new attention to them.¹²⁰

For Lakoff, novel conceptual metaphors are new cognitive mappings that arise through human creativity and imagination.¹²¹ It is important to note that “novel” opposes “conventional” in CMT and is thus a technical term that extends beyond mere chronology. Because conventional metaphors are usually utilized unconsciously, automatically, and effortlessly, it seems plausible that a use of conscious effort would be a defining characteristic of novel metaphor.¹²² A few other features of this understanding of novel metaphor deserve mention. Novel conceptual metaphors may, but need not, lead directly to a novel linguistic realization; the metaphor may be implemented experientially without being realized expressively. It is also plausible that, rarely, an accidentally or randomly generated novel utterance may induce a new conceptual metaphor by serving as an experiential basis, not entirely unlike when someone makes an unintended pun that is only later recognized as such. It is an important feature of the way concepts connect to language — including the language of mathematics — that sentences may be created which satisfy the formal syntactic rules but whose conceptual significance or meaning is only later appreciated.¹²³ Regardless of how they arise, novel

¹²⁰One may argue that substantial extension to a conceptual metaphor seems more akin to genuine novelty than it does to minimal, straightforward extension. However, my purpose here was merely to reemphasize the distinction between linguistic and conceptual metaphor and to note that linguistic novelty may, but need not, be accompanied by conceptual novelty. Therefore, I will not address this argument.

¹²¹Lakoff and Johnson, *Metaphors We Live By* 139.

¹²²Lakoff, “Contemporary Theory of Metaphor” 245. The CMT understanding of novelty versus conventionality emphasizes the individual rather than the population; it is important to remain cognizant of the context in which these adjectives are being applied in order to avoid error.

¹²³A related, more significant case has been discussed briefly above: a speaker may utter a sentence invoking a conceptual metaphor that their audience does not possess. Such a sentence would be an instantiation of a

conceptual metaphors are typically fleeting. Some are adopted into the collective conceptual system, having their gleam of novelty worn to a patina of conventionality. Others vanish into oblivion, lacking sufficient interest or underlying conceptual support to persist.¹²⁴ In theory, a conceptual metaphor may be compelling and useful, and yet strange enough to defy conventionalization, thereby remaining novel for an extended duration.

This raises the question of whether *all* conventional metaphors begin as novel metaphors in the sense given in the previous paragraph. It is clear that every conceptual metaphor must have been *new* at some point: I possessed no conceptual metaphors before I was born, and there were no conceptual metaphors at all before life came into existence. However, if novelty is understood to involve conscious creativity then some conceptual metaphors can never have been novel in this sense. In particular, most primary metaphors cannot be novel insofar as they arise in humans in infancy, arguably prior to the development of consciousness.¹²⁵ On the other hand, the typically one-off nature of image metaphors suggests that they will frequently be novel, though there are many cases where image metaphors have been conventionalized and entered our lexicon, such as “computer mouse.” It is difficult to do more than speculate about whether particular conventionalized complex metaphors started their lives as novel metaphors or were “metaphors stillborn.”¹²⁶ For example, while it seems clear that *LIGHT IS A WAVE* originated as a novel metaphor created by physicists in the seventeenth century, it is not obvious if *ARGUMENT IS WAR* was ever novel.¹²⁷ All of this suggests that there are at least two ways conceptual metaphors may be born: either they are the conscious, intentional, and relatively abrupt product of an individual, or they

metaphor for the speaker and, potentially, part of the experiential basis of a novel (to them) metaphor for the audience.

¹²⁴If the human brain is constantly seeking to form new connexions, it is plausible that the vast majority of novel conceptual metaphors are untenable and are abandoned even before they reach the conscious mind. Random, non-systematic juxtapositions of concepts — like *FURNITURE IS CHEESE* — are typically entirely untenable — or at least practically useless. Such speculative hypotheses must be tested by neuropsychological experimentation that is beyond the scope of this dissertation.

¹²⁵Lakoff and Johnson, *Metaphors We Live By*, 256: “Primary metaphors arise spontaneously and automatically without our being aware of them.”

¹²⁶Quine 160.

¹²⁷While it is desirable to use original examples in academic work whenever possible, it is hopefully clear why I am relying on canonical conceptual metaphors here. It would take an unreasonable amount of time and effort to uncover a common conventional metaphor that Lakoff and his cohort have not already catalogued, and creating a novel conceptual metaphor that is genuinely new, tenable, and that serves as a good illustrative example would require a stroke of creative genius that has not yet struck.

develop gradually, surreptitiously, and communally and are conventional from conception.¹²⁸ The idea that much of conceptual development is unconscious and gradual while some is conscious and abrupt seems relatively uncontroversial yet has important consequences for any concept acquisition story. One important and controversial claim made by CMT is that new conceptual metaphors may actually *create* similarities between concepts.

Chapter 2 looked at popular comparison theories of metaphor which hold that metaphors are elliptical similes, comparisons of the salient similarities between two things. Comparisons cannot create similarities, they can only pick out pre-existing, isolated resemblances. In CMT, mainly image metaphors function in this way. Other conceptual metaphors are “typically based on cross-domain correlations in our experience, which give rise to the perceived similarities between the two domains within the metaphor.”¹²⁹ Thus, while two things may superficially resemble each other, any conceptual metaphor connecting them will be based on deeper experiential correlations. The creation of similarity is governed by Lakoff’s hypothesized Invariance Principle: “Metaphorical mappings preserve the cognitive topology (that is, the image-schema structure) of the source domain, in a way consistent with the inherent structure of the target domain.”¹³⁰ The basic idea is this: concepts have varying amounts of inherent structure emerging directly from experience, typically in the form of image schemata. The posited inference preserving nature of conceptual metaphors requires that if an image schema is mapped from the source domain then it must be aligned with the structure of the target domain; the inherent structure of the target domain constrains the mapping possibilities. However, it was above noted that conceptual metaphors tend to be directed from the less abstract to the more abstract. Abstract concepts such as LOVE possess relatively little inherent structure. This has two important, related consequences. First, this lack of structure leaves us with few resources for understanding and reasoning about our many and varied experiences of love; the inherent structure of love seems to include only an

¹²⁸The relationship between novelty and conventionality, particularly with respect to transitions between the two as well as considerations of conceptual metaphors in individuals versus populations, is one place that CMT could use some improvement. Cornelia Müller makes some positive suggestions in this direction which are discussed below.

¹²⁹Lakoff and Johnson, *Metaphors We Live By* 245.

¹³⁰“Contemporary Theory of Metaphor,” 215. It should be noted that in the 2003 afterword to *Metaphors We Live By*, Lakoff describes the Invariance Principle as “ugly” and “unfortunate,” and cites this as motivation for moving to a neural theory of language (254).

impoverished skeleton of a concept: “a lover, a beloved, feelings of love, and a relationship, which has an onset and often an end point.”¹³¹ Second, their lack of structure means that abstract concepts have fewer limitations arising from the Invariance Principle. This absence of inherent structure can be seen as a potential for taking on new structure via conceptual metaphor.¹³² These two consequences both motivate and allow for the metaphoric importation of structure from less-abstract concepts to aid our understanding. This importation is not unlike filling in details gleaned from the world on a blank section of a terrain map. It is this imposition of new structure, the creation of target entities where there once was void, that underlies the creation of similarity claim.¹³³ The controversy surrounding the creation of similarity claim will be discussed in further detail below. A third important consequence of the relative lack of inherent structure is that a single abstract concept can serve as the target domain for multiple conceptual metaphors.

The less inherent structure a concept possesses, the greater potential it has for becoming structured via conceptual metaphor. As mentioned above, LOVE has very little inherent structure and so people’s many and varied experiences of love tend to be understood using conceptual metaphors. Because experiences of love can and do vary drastically, a variety of metaphors are employed, including LOVE IS A JOURNEY, LOVE IS NOURISHMENT, LOVE IS MADNESS, and LOVE IS WAR.¹³⁴ Each of these metaphors emphasizes and explains particular aspects of love experiences while masking others. LOVE IS WAR, for example, provides structure for understanding experiences of love involving conflict and injury but does little to explain more cooperative aspects. Thus, abstract concepts will often be structured by a patchwork of several conceptual metaphors:

Each mapping is rather limited: a small conceptual structure in a source domain mapped onto an equally small conceptual structure in the target domain. For a rich and important domain of experience like love, a single conceptual mapping does not do the job of allowing us to reason and talk about the experience of love as a whole. More than one metaphorical mapping is needed.¹³⁵

¹³¹Lakoff and Johnson, *Philosophy in the Flesh* 70.

¹³²It should be noted that even concepts displaying thorough inherent structuring may take on further structure by way of conceptual metaphor.

¹³³Lakoff and Johnson, *Metaphors We Live By* 253. It is worth noting the connexion between the creation of similarity and the discussion of novel linguistic metaphors extending conceptual metaphors above.

¹³⁴Lakoff and Johnson, *Metaphors We Live By* 141.

¹³⁵Lakoff and Johnson, *Philosophy in the Flesh* 71.

There are several ways metaphors with a shared target domain may relate to one another. If the experiences associated with a certain concept are sufficiently diverse, incompatible or even outright contradictory metaphors may provide structure to the concept. When we consider that incompatible attributes — such as evenness and oddness — frequently belong to the same concept, the idea of incompatible metaphors structuring a concept becomes less objectionable; as long as they are not invoked simultaneously, there is no problem. However, when incompatible conceptual metaphors are invoked within a single thought or sentence, an undesirable dissonance arises that explains the traditional prohibition against mixing one’s metaphors.¹³⁶ Not all conceptual metaphors are incompatible, and CMT explains that it can be advantageous to layer or combine metaphors. For example, we have already seen how primary metaphors can positively interact to constitutively ground a complex metaphor. Lakoff and Johnson sometimes refer to such strongly interacting primary metaphors as *consistent*. By contrast, metaphors that positively interact but whose structural overlap is less complete and integrated are said to be *coherent*. Metaphoric coherences will typically involve the source domains having common or connected image-schematic structure.¹³⁷ For example, the metaphors AN ARGUMENT IS A JOURNEY and AN ARGUMENT IS A CONTAINER may be used constructively in the same sentence to describe our experience of an argument as having a direction and a content; neither of these metaphors alone captures both of these aspects. One of the reasons these metaphors play well together is that the PATH schema and the CONTAINER schema are spatial schemata that can be coherently superimposed: we may follow a path through a contained space, for example. Such superimpositions can lead to shared entailments, as illustrated by the sentence “Now that he’s halfway through his argument, I can see where he’s going.”¹³⁸ Though Lakoff seems to have abandoned the consistency versus coherency terminology early on, the idea that metaphors can interact in different ways remains legitimate and useful; in particular, we shall see in chapter 4 that Lakoff and Núñez claim that four interacting coherent metaphors collectively serve as the foundational ground of arithmetic.¹³⁹ The idea that metaphorical mappings work in concert to help bring conceptual structure to

¹³⁶For an extended, positive discussion of mixed metaphors, see Müller, chapter 5.

¹³⁷Lakoff and Johnson, *Metaphors We Live By* 95.

¹³⁸Lakoff and Johnson, *Metaphors We Live By* 92, 103.

¹³⁹See *Where Mathematics Comes From*, chapter 3.

our experiences naturally leads us to question if other cognitive mechanisms help form the orchestra, as it were.

Linguists and rhetoricians have traditionally distinguished a wide variety of linguistic tropes — figures of speech or, to be more etymologically faithful, *turns* of phrase: irony, simile, metaphor, hyperbole, meiosis, metonymy, synecdoche, litotes, alliteration, and tmesis, to name but a few.¹⁴⁰ Many tropes, such as alliteration, bear little resemblance to metaphor and seem more purely linguistic rather than constitutively conceptual, being typically used to embellish, enliven, or emphasize. Aristotle already noted this difference between kinds of tropes in his *Poetics*.¹⁴¹ However, as mentioned in chapter 2, simile, metonymy, synecdoche, and analogy do seem relevantly similar to linguistic metaphor and therefore warrant consideration, as a way of clarifying the idea of metaphor by exploring the details of its boundary. Lakoff seems to endorse such an approach when he says “[t]o understand what is metaphorical, we must begin with what is not metaphorical. In brief, to the extent that a concept is understood and structured on its own terms — without making use of structure imported from a completely different conceptual domain — we will say that it is not metaphorical.”¹⁴² Lakoff and Johnson have posited conceptual metaphor as a feature of cognition that explains a variety of phenomena, linguistic and otherwise. The question thus arises, does conceptual metaphor explain these apparently related linguistic tropes or are they each associated with distinct conceptual mappings?

Metonymy is a less familiar trope than metaphor; many people will have never heard of it even though they employ it. Those who are acquainted with the term likely describe it as a figure of speech wherein one word or phrase is substituted for another based upon an association between the two; the association involved is usually one of contiguity, with the substituted term being a property of, a symbol for, or elsehow connected to the replaced

¹⁴⁰While most of these tropes will be familiar or explicitly discussed below, some are less well known. Meiosis is conspicuous understatement; for example, referring to World War II as ‘a skirmish.’ Notably, A.P. Martinich claims that several canonical examples of linguistic metaphor — such as “No man is an island” — are actually instances of meiosis (“A Theory for Metaphor” 454–5). Litotes involves a sort of double negation, as in endorsing someone’s position by telling her “You’re not wrong.” Tmesis occurs when a word is bifurcated, frequently so that another word may be inserted as an infix, as in “unbloodylikely” and “absofuckinglutely.”

¹⁴¹*Poetics*, 1457b.

¹⁴²Lakoff and Turner 57.

term.¹⁴³ An example of a linguistic metonymy is “I enjoy reading Plato,” where “Plato” stands for “the writings of Plato.” From its inception, CMT has held a view of metonymy similar to its view of metaphor: metonymic and synecdochic sentences are merely symptoms of underlying conceptual metonymies.¹⁴⁴ For example, the example above employs the conceptual metonymy CREATOR FOR CREATION. However, conceptual metonymy is a distinct kind of cognitive mapping from conceptual metaphor.¹⁴⁵ Whereas metaphor is a cross-domain inferential mapping, metonymy is an intradomain referential mapping: “in a *metonymy*, there is only one domain: the immediate subject matter. There is only one mapping: typically the metonymic source maps to the metonymic target (the referent) so that one item in the domain can stand for the other.”¹⁴⁶ Metaphor is conceived as necessarily involving the crossing of a gap between the source domain and target domain, the use of one thing in thinking about another, distinct thing. By contrast, metonymy requires that such a gap necessarily *not* exist, that the source entity and target entity be connected and “form a single, complex subject matter.”¹⁴⁷ Thus, conceptual metaphor and metonymy must be distinct mechanisms, as one necessarily involves bridging a gap that is essentially absent in the other: “metonymy [is] a *stand-for relationship* between two elements within a single conceptual domain and metaphor [is] an *is-understood-as relationship* between two conceptually distant domains.”¹⁴⁸ A metonymy can map between seemingly distinct domains, such as TIME and SPACE, as long as it exploits a pre-existing connexion that yields a single complex subject matter. For example, conventionalized, contextual understandings of rates of travel allow for the TIME FOR DISTANCE metonymy underlying the sentence “Regina is two-and-a-half hours from Saskatoon.” Notice that in this metonymy there is no transfer of logic or structure between the two domains as there is in the TIME IS SPACE metaphor: it is only the nature of the pre-existing relationship

¹⁴³“metonymy, n.,” *OED Online*, Sept. 2011, Oxford UP, 22 Sept. 2011.

¹⁴⁴In many accounts of figurative language, synecdoche is taken to be a special instance of metonymy. For CMT, linguistic synecdoches are symptoms of the conceptual metonymy THE PART FOR THE WHOLE (Lakoff and Johnson, *Metaphors We Live By* 36).

¹⁴⁵Though they are distinct kinds of mapping, metonymy and metaphor frequently interact in the creation and interpretation of figurative sentences. Moreover, it has been plausibly suggested that conceptual metaphors may sometimes be motivated by or derived from conceptual metonymies (Kövecses 157).

¹⁴⁶Lakoff and Johnson, *Metaphors We Live By* 265; emphasis theirs.

¹⁴⁷Lakoff and Johnson, *Metaphors We Live By* 267.

¹⁴⁸Kövecses 227; emphasis his.

connecting the two domains that is of interest.¹⁴⁹ CMT holds that, like metaphor, “metonymy is one of the basic characteristics of cognition.”¹⁵⁰ Lakoff claims that prototype effects are often the result of metonymic mappings that substitute either a category member or a subcategory for an entire category; CMT uses conceptual metonymy to explain intraconceptual graded structure and privileged category elements, including prototypes, stereotypes, ideals, paragons, and generators.¹⁵¹ Indeed, combining the commonplace and CMT understandings of metonymy suggests that symbolic thought and symbol use in general — including our linguistic capabilities — may involve conceptual metonymy insofar as we substitute symbols for the concepts they are associated with; Lakoff and Turner note the existence of a WORDS STAND FOR THE CONCEPTS THEY EXPRESS metonymy, though their discussion is dismayingly brief given the important implications of this mapping.¹⁵² While conceptual metonymy is required to explain both linguistic metonymies and much of our conceptual lives, there is no such thing as conceptual simile.

While conceptual metaphor cannot explain linguistic metonymy or synecdoche, it does seem to be integral to the creation and interpretation of many linguistic similes. As mentioned in chapter 2, similes are usually understood as figurative sentences containing comparisons that use either “like” or “as.” Identifying such sentences as “figurative” indicates an essential dissimilarity between the things being compared. That is, most people would be disinclined to classify comparisons such as “Sex is like sex,” “My hand is like your hand,” or “A kilt is like a skirt” as similes, whereas two prototypical examples of simile are “She is as slow as a tortoise” and “My car is like a member of the family.” Leaving the “like” out of the last example transforms it into a linguistic metaphor: “My car is a member of the family.” This close connexion between the two tropes led to the comparison theory of metaphor and its central belief that all metaphors are elliptical similes. Lakoff and Johnson argue that CMT is incompatible with the comparison theory of metaphor because its understanding of the similarities involved in simile as isolated and preexisting is in conflict with the metaphoric

¹⁴⁹Lakoff and Johnson, *Metaphors We Live By* 266.

¹⁵⁰Lakoff, *Women, Fire, and Dangerous Things* 77.

¹⁵¹*Women, Fire, and Dangerous Things* 90.

¹⁵²Lakoff and Turner 108.

systematicity and the creation of similarity that are integral to their position.¹⁵³ While some metaphors (in particular, image metaphors) may merely highlight preexisting perceptual similarities, it seems necessary that something more is going on in those involving more abstract domains; for example, the interpretation of “I need a moment to digest what you’ve said” seems to require structural inferences that go beyond the necessarily limited perceptual resemblances between concrete foods and abstract thoughts.¹⁵⁴ This argument is apparently the only place Lakoff explicitly discusses simile but, conveniently, the absence of discussion itself serves as evidence that simile is not of central concern to CMT. In particular, the entire lack of discussion implies that there is neither evidence nor theoretical need for “conceptual simile.” Since metaphor is not reducible to simile in CMT, and there is no distinct cognitive mapping underlying simile, this suggests that similes may be explained by conceptual metaphor, a suggestion that is supported by the fact that simile often involves understanding one thing in terms of another.¹⁵⁵ Whereas attempted definitions of linguistic metaphor tend to be problematic and controversial, the definition of simile in terms of its linguistic construction is straightforward, clear, and widely accepted; metaphorical sentences can come in an infinity of different varieties while similes are more limited in form. While it may be true that similes are often best accounted for by conceptual metaphor, the details of the explanation still need filling in, including a discussion of the ramifications of the explicit nature of simile and the apparently vacuous truth and bidirectionality of “like” statements; further research in these directions and others may expose simile as a richer and more complex and diverse phenomenon than is generally believed. As these details are entirely tangential to the present line of inquiry, I will pursue this no further.

The above paragraph raises a final important question: why does Lakoff refer to cross-domain cognitive mappings as metaphors? The argument in the previous paragraph explains why it makes sense to refer to the cognitive mappings underlying both metaphor and simile as conceptual metaphors rather than conceptual similes. However, more should be said about why these maps are called metaphors at all. The terminology originally arises from the observation that clusters of systematically related metaphorical sentences can be understood

¹⁵³ *Metaphors We Live By* 153.

¹⁵⁴ Kövecses 69.

¹⁵⁵ Lakoff and Turner 57.

as symptoms of an underlying cognitive structure. Referring to the underlying cause as *the* metaphor makes sense in the same way that referring to a virus as measles does. By defining metaphor as a kind of cognitive mapping rather than a linguistic device, CMT sidesteps or dissolves several difficulties that vex traditional views such as the identification problem. In particular, CMT seems well equipped to handle metametaphors, that is, metaphors used to comprehend the abstract concept METAPHOR. While the common — and arguably unavoidable — practice of using metaphor to understand metaphor is problematic on many traditional accounts, it counts as a point of evidence in favour of CMT: “Every scientific theory is constructed by scientists — human beings who necessarily use the tools of the human mind. One of those tools is conceptual metaphor. When the scientific subject matter is metaphor itself, it should be no surprise that such an enterprise has to make use of metaphor.”¹⁵⁶ On the other hand, the fact that “contemporary metaphor theorists commonly use the term ‘metaphor’ to refer to the conceptual mapping”¹⁵⁷ has led to confusion and disagreement insofar as many people still think of metaphor as a primarily or exclusively linguistic phenomenon. A certain amount of controversy and criticism might have been avoided by adopting an alternative term that did not lend itself to unfortunate equivocation.¹⁵⁸ For example, to at least some degree, “conceptual metaphor” seems synonymous with “analogical thought.”¹⁵⁹ As the practice of referring to cross-domain cognitive mappings as (conceptual) metaphors is now over 30 years old and well established, I will adopt this terminology but try to use it carefully to avoid equivocating misreadings. However, I am entirely interested in the cognitive mechanism itself and not particularly concerned about how it is labeled: a cognitive metaphor by any other name would work as sweetly.

To sum up, Lakoff and his collaborators hold that conceptual metaphors are cognitive mechanisms that make a fundamental contribution to the human experience. They are neural mappings between conceptual domains that structure the abstract by preserving inferences mapped from an ubiquitous and basic sensorimotor schematic ground. Linguistic metaphors

¹⁵⁶Lakoff and Johnson, *Metaphors We Live By* 252.

¹⁵⁷Lakoff, “Contemporary Theory of Metaphor” 209.

¹⁵⁸Mathew S. McGlone, “What is the explanatory value of a conceptual metaphor?,” *Language and Communication* 27.2 (2007): 111.

¹⁵⁹See, for example, Douglas Hofstadter’s extensive writings on analogy.

are just one of many symptoms of the systematic network of conceptual metaphors — a network that is also utilized in our thought and the physical creations and institutions arising therefrom. The idea that metaphors are a matter of thought rather than language is not as novel as Lakoff's writings sometimes intimate; however, the advent of making conceptual metaphor the central hub of an ongoing programmatic inquiry into the nature of language and thought was a revolutionary move that helped his view achieve more success than its predecessors. CMT has several major strengths. By understanding metaphor as primarily conceptual and only derivatively linguistic, it bypasses many of the problems plaguing traditional linguistic views that were discussed in chapter 2. CMT's commitment to empirical responsibility means that scientific evidence permeates the theory and lends strength to the supporting arguments. CMT is both parsimonious and fecund, in that conceptual metaphor and a few other cognitive mechanisms do an enormous — and ever-increasing — amount of explanatory work. Despite its various strengths, the Lakovian position has many objectors and has not yet found widespread acceptance in the philosophy of language, nor in the academy generally.

3.3 Criticisms and Objections

Distilling thirty years of evolving CMT canon into a précis is a difficult task. Definitively addressing the voluminous and diverse body of criticism and debate surrounding this controversial theory seems an insurmountable one. Rather than attempt to consider every criticism of CMT, the final section of this chapter will focus on a handful of objections that are pervasive in the literature or that are otherwise significant. It is obvious that conceptual metaphor theory has serious implications for how one understands concepts. Prinz's desiderata will be used to expose strengths and weaknesses in this understanding, and thereby fend off misguided objections while identifying others as potentially more telling.¹⁶⁰ Further structure for this section of the chapter comes from Leezenberg's section on Lakoff's theory in his

¹⁶⁰It should be noted that a commitment to conceptual metaphor does not necessarily exhaustively determine a theory of concepts; that is, more than one theory of concepts may be compatible with conceptual metaphor. Thus, even if the particular understanding of concepts discussed below turns out to be fatally flawed, this does not necessarily require one to abandon the notion of conceptual metaphor.

Contexts of Metaphor, a rogue’s gallery of noteworthy objections. I will explain why several of common philosophical objections to CMT are misguided, and suggest ways that the theory may develop to overcome some of the more telling criticisms. It is worth remembering at this point that the main reason for this defense of CMT is to allow discussion of Lakoff and Núñez’s theory of mathematics in chapter 4 to be relevant. Thus, it is not essential that we commit to CMT wholesale, only that we allow that metaphor is not a purely linguistic phenomenon and that at least some mathematical concepts could be constitutively conceptualized metaphorically.

Before moving on to the philosophical objections, it should be noted that several authors have criticized Lakoff’s scholarship. Though many of the books in the CMT canon are explicitly written for a nonspecialist audience, the near complete absence of in-text citations is troubling; Jackendoff and Aaron observe that “[t]his compromise between the needs of informal exposition and scholarly discourse strikes us as unsatisfactory.”¹⁶¹ Additionally, *Metaphors We Live By* and *More Than Cool Reason* have limited bibliographies with less than 20 entries apiece that conspicuously fail to include important works on metaphor that are being argued against. Max Black notes, rightfully, that *Metaphor We Live By*’s “absence of an index is deplorable.”¹⁶² These factors combine to make CMT scholarship — such as the summary provided above — needlessly difficult and frustrating. Moreover, Lakoff and his coauthors seem strangely reluctant to engage in exchanges with their peers, rarely acknowledging or addressing even constructive criticisms.¹⁶³ Lakoff’s scholastic style thus diverges from the traditional *modus operandi* of the academy, where transparent, thorough referencing and interactive debate and criticism among peers is the norm. In addition to being put off by Lakoff’s unconventional scholarship, some readers may have been incensed

¹⁶¹Ray Jackendoff and David Aaron, “Review of *More Than Cool Reason*,” *Language* 67.2 (1991): 321.

¹⁶²Max Black, “Review of *Metaphors We Live By*,” *J. of Aesthetics and Art Criticism* 40.2 (1981): 210. It seems that Lakoff may have taken these criticisms to heart: *Philosophy in the Flesh* has 18 pages of categorized references and a 22-page index, and *Where Mathematics Comes From* has 20 pages of each. Unfortunately, in-text citations remain scant in more recent works.

¹⁶³I am aware of only two instances where Lakoff has provided an explicit defense against one of his critics. One is his well-known response to Steven Pinker’s book review of *Whose Freedom?: The Battle over America’s Most Important Idea*. The other is a paper coauthored with Mark Johnson responding to Martina Rakova’s criticisms of their philosophy. Lakoff’s response is that many of the criticisms leveled depend on assumptions that are called into question by CMT’s empirical research and that the critic fails to address this incompatibility (Johnson and Lakoff, “Why cognitive linguistics requires embodied realism” 260).

by the sometimes inflammatory and partisan political theorizing that has become one of his main foci over the last decade. While none of these criticisms are fatal to Lakoff's theory, they do make a contribution to the general apprehensiveness towards CMT found in much of the literature.

As I mentioned at the beginning of the chapter, many of the philosophical criticisms of CMT cluster around CONCEPT and fall into two categories. The first category involves the charge that conceptual metaphor entails an unsatisfactory understanding of concepts. Included in this category are objections that agree metaphor is a matter of thought, but hold that Lakovian conceptual metaphor does not sufficiently explain the phenomena in question. To defend against these criticisms, I will evaluate CMT using Prinz's seven desiderata for theories of concepts, and suggest ways in which its weaknesses may be overcome. I am not particularly concerned with the subcategory of criticisms that reject the details of conceptual metaphor because they concur that thought is metaphoric, which is the result I primarily wish to defend. The second category of criticisms focuses on the connexion between concepts and language in CMT. As Prinz intentionally excluded language desiderata from his list, we shall have to look elsewhere for support. The work of Cornelia Müller takes an approach that could help overcome these concerns.

In CMT, "*concepts* are neural structures that allow us to mentally characterize our categories and reason about them."¹⁶⁴ Categories are not mental representations of natural kinds gleaned from the world, "sets defined by common properties of objects."¹⁶⁵ Rather, Lakoff says that "human categorization is essentially a matter of both human experience and imagination — of perception, motor activity, and culture on the one hand, and of metaphor, metonymy, and mental imagery on the other."¹⁶⁶ His embodied-realist theory of conceptual categories holds that thought, experience and, therefore, categorization essentially involve an *interaction* between an organism and its environment.¹⁶⁷ Though Lakoff extensively develops this theory of categorization in *Women, Fire, and Dangerous Things*, delving into it would be a lengthy, distracting tangent. Therefore, for the sake of simplicity and generality, I will

¹⁶⁴Lakoff and Johnson, *Philosophy in the Flesh* 19; emphasis theirs.

¹⁶⁵Lakoff, *Women, Fire, and Dangerous Things* 10.

¹⁶⁶*Women, Fire, and Dangerous Things* 8.

¹⁶⁷Johnson and Lakoff, "Why cognitive linguistics requires embodied realism" 249.

restrict my discussion as much as possible to aspects of the theory immediately connected with conceptual metaphor as discussed above.

As a theory of concepts, CMT has several strengths. CMT does an excellent job of satisfying the scope desideratum. The variety of cognitive structures and mappings it includes — including, but not limited to, conceptual metaphor, conceptual metonymy, and image-schemata — can account for a wide range of conceptual kinds and levels of abstraction. Should a concept outside of CMT’s scope be found, CMT is well positioned to rectify its deficiency by incorporating new cognitive mechanisms; conceptual metaphor plays well with a wide variety of conceptual mechanisms and structures. In *Women, Fire, and Dangerous Things*, Lakoff puts forward a theory of concepts as *idealized cognitive models*, and several chapters are devoted to discussing and accounting for a variety of different models — including cluster models, radial models, and classical definitional models.¹⁶⁸ The particulars of Lakoff’s theory of categorization aside, any account of concepts that can accommodate conceptual metaphor will include the idea that concepts have differing kinds and amounts of structure, and will therefore be better able to satisfy the scope desideratum than the classical theory of concepts.

CMT also does well with the categorization desideratum. Lakoff frequently emphasizes that a core value of his project is having his theories account for or be consistent with the body of experimental evidence.¹⁶⁹ This commitment to being empirically responsible is very apparent in *Women, Fire, and Dangerous Things*, which “surveys a wide variety of rigorous empirical studies of the nature of human categorization” and integrates these results into CMT.¹⁷⁰ In particular, Lakoff emphasizes the importance of Rosch’s experimental findings on prototype effects and basic-level categories in ushering in a new paradigm in concept scholarship. Lakoff holds that conceptual metonymy can explain Rosch’s results: a metonymy is invoked when a person thinks or reasons about a category in general by means of a particular prototypical representative, for example.¹⁷¹ Despite Lakoff’s emphasis on Rosch’s typicality evidence in the development of his theory, CMT is not an instance of a

¹⁶⁸ *Women, Fire, and Dangerous Things* 153–4.

¹⁶⁹ For example, see Lakoff and Johnson, *Metaphors We Live By* 246.

¹⁷⁰ Lakoff, *Women, Fire, and Dangerous Things* 11.

¹⁷¹ *Women, Fire, and Dangerous Things* 79.

prototype theory as it rejects the idea that categories are defined by prototypes.¹⁷² Note that avowing empirical responsibility does not necessarily mean that one's theory actually accounts for all the relevant facts; critics could object that CMT does not adequately account for the empirical facts it claims to, or that it fails to take some particular piece of relevant evidence into account.¹⁷³ In general, such empirical criticisms seem more likely to suggest directions for further development of the theory than to be devastating. Addressing questions of the empirical adequacy of CMT is beyond the scope of my dissertation; a robust treatment of the relevant scientific results would require a much-larger, book-length project.¹⁷⁴

The summary of conceptual metaphor presented above suggests that CMT satisfies the acquisition and publicity desiderata: concepts emerge from recurrent patterns in our experiences, and are public because those experiences involve our being similarly embodied in a shared environment. One of the key strengths of CMT is that its emphasis on cognitive mappings naturally provides an account of conceptual emergence and development, an aspect that is often neglected by other philosophical theories. The account of acquisition provided by Lakoff seems quite plausible, if complex and somewhat lacking in detail. I will therefore devote my attention to claims that the publicity desideratum may not be successfully met. Prinz argues that theory-theory fails to satisfy publicity because “[i]t is very unlikely that any two people have exactly the same theories of the categories they represent... Theories mushroom out to include all our beliefs about a category. Such large belief sets inevitably differ from person to person.”¹⁷⁵ One may argue that Lakovian idealized cognitive models are enough like theories to fall victim to this criticism as well. However, the constraints imposed by embodiment mean that idealized cognitive models cannot be as problematically ad hoc and idiosyncratic as Prinz claims theories are.¹⁷⁶ Moreover, while successful communication requires that we possess *shared* concepts to some degree, this does not entail that everyone's concepts must be *identical*. Prinz considers this response — that conceptual similarity is sufficient for publicity so strict identity is unnecessary — and concludes that it is not in the spirit

¹⁷² *Women, Fire, and Dangerous Things* 136–7.

¹⁷³ See McGlone (2007) for an example of a criticism of the second type.

¹⁷⁴ See Gibbs (2011) for a recent defense of CMT from the perspective of a cognitive scientist.

¹⁷⁵ Prinz 87.

¹⁷⁶ Prinz 87.

of theory-theory. However, such an approach does seem to be consistent with, and therefore available to, both prototype theory and CMT.¹⁷⁷ Concepts differ between cultures as well as between individuals and can cause communicative difficulty or even outright communicative failure. One place that the effects of conceptual disagreement can be observed is in discussions about abstract notions, such as the afterlife. All of this suggests that Prinz's original formulation of the publicity desideratum may be too strong, a conclusion he endorses insofar as the defense of his proxytype theory of concepts utilizes a relaxed version of the publicity requirement that does not require strict identity between concepts.¹⁷⁸ Thus, like proxytype theory, CMT satisfies the publicity desideratum better than theory-theory insofar as it allows for variation among individual's concepts but commonalities of embodied experience provide an intersubjective core to many concepts that allows for successful communication.

Michiel Leezenberg explicitly criticizes Lakoff for failing to account for conceptual publicity: he claims if conceptual metaphors precede their linguistic instantiations as CMT claims then it is difficult to understand how "*private* concepts or experiences warrant that people have the same *public* meanings."¹⁷⁹ In particular, Leezenberg is worried that CMT contains a vicious circularity: pre-conceptual structure grounds conceptual structure, which allows for language, which is necessary for culture, which plays an essential role in pre-conceptual structure.¹⁸⁰ While Leezenberg's observation that Lakoff could spend more time considering the details of how cultural factors contribute to concepts is well taken, his argument that CMT fails to meet the publicity requirement seems problematic. He erroneously assumes that concepts must be wholly structured prior to language use.¹⁸¹ The very idea that a

¹⁷⁷Prinz 88–9.

¹⁷⁸Prinz 158. After using his desiderata to establish the faults of various popular theories of concepts, Prinz puts forward a theory of concepts as proxytypes, perceptually derived long-term memory networks that are currently, or have the potential to be, activated in working memory as a mental representation of a category (148–9). Unlike prototype theory and theory-theory, which identify conceptual structure as a single kind of information, proxytype theory insists that multiple kinds of knowledge are involved in concept formation (Prinz 314). Prinz also emphasizes that the representations necessarily retain their perceptual modality (119). Thus we see that there are parallels between proxytypes and embodied idealized cognitive models. Though Prinz and Lakoff do not mention these parallels, Lawrence Barsalou (whose work underpins proxytype theory) explicitly acknowledges similarities between elements of his theory and Lakovian idealized cognitive models (Barsalou 17). Reconciling Prinz and Lakoff on concepts could be fruitful, but the constraints of this dissertation necessitate that such an undertaking happen elsewhere.

¹⁷⁹Leezenberg 141; emphasis his.

¹⁸⁰Leezenberg 142. A similar criticism occurs in Rakova (2002).

¹⁸¹Leezenberg 142.

concept can be completely structured is problematic, as concepts are dynamic and have the potential for adaptive development prompted by new experiences: humans learn! The dynamic interplay between conceptual metaphors, their experiential bases, and their cultural and linguistic realizations that was discussed above suggests that the circularity that Leezenberg refers to is virtuous rather than vicious.¹⁸² Recall the discussion about circles (and other regular polygons): geometric regularities in the environment are experienced, those experiences lead to a primitive conceptualization of CIRCLE, that conceptualization allows people to create representations of circles and to talk about them, and thus more circles and circle-talk are introduced into the environment, the experience of which allows for further development of the CIRCLE concept, and so on. Such virtuous conceptual circles exist at both the individual and population levels, and they interact: pre-conceptual infants are born into an environment pervaded with both linguistic and non-linguistic instantiations of concepts of the humans who have come before them. In this sense, language can precede concepts. For many concepts, a cultural contribution occurs only after a substantial conceptual core has developed based on common biological experience. That is, while culture may indeed play an important role in pre-conceptual structure as Leezenberg claims, it is clear that organisms possess a variety of discriminatory mechanisms that also provide significant pre-conceptual structure but are not culture dependent. Leezenberg's philosophical arguments fail to take into account the body of evidence Lakoff appeals to in suggesting that commonalities in the embodied experiences of infants provides the ground for the public intersubjectivity of concepts. Moreover, his concern that "the mere fact that two individuals live in the same environment, and behave in the same way, gives no conclusive evidence as to their inner workings"¹⁸³ seems overly skeptical; while this observation has some merit, for our purposes it can be adequately addressed by the theoretical virtues of simplicity and parsimony. Thus,

¹⁸²It may be fruitful to conceive of virtuous circularity as a helix rather than a circle, progressing upward as it goes around rather than problematically returning to its origins. Another related way of conceiving a virtuous circle that may be useful relies on the distinction between displacement and distance. If one circumnavigates the Earth and ends at the same place they started, then their displacement is zero: rather than traveling from point A to point B, they have traveled from point A to point A and haven't "gotten anywhere." However, even if one ends up where they started, the distance covered going around the circle can be extensive. Just as repeatedly traversing a circular trail can etch a path into the grass, going around conceptual circles need not be an exercise in futility but rather can lead to conceptual development and refinement.

¹⁸³Leezenberg 141.

while the details of CMT’s complex story of concept acquisition and grounding could use further elaboration, it seems plausible that it satisfies the publicity desideratum given the current state of the theory.

Whether CMT satisfies the compositionality criterion is debatable, largely because the desideratum itself is somewhat contentious. Following Fodor, Prinz states that “[c]oncepts are compositional just in case compound concepts (and thoughts) are formed as a function of their constituent concepts together with rules of combination.”¹⁸⁴ However, as was the case with publicity, Prinz claims there are reasons to believe that this formulation of the compositionality requirement is too stringent. In particular, he says that the desideratum should be interpreted as saying that a theory must *allow* for conceptual composition but that it need not require that the entire content of a concept *must* be inherited from its constituents.¹⁸⁵ When relevant background knowledge or experience with exemplars of a complex concept are available, we should expect that such knowledge will often trump pure compositional rules in providing structure.¹⁸⁶ For example, most people’s understanding of GREEN APPLE includes the notion TARTNESS, a notion that belongs to neither APPLES nor GREEN but is based on their experiences of eating green apples. Prinz concludes that, contra Fodor, both prototypes and proxytypes are compositional in this relaxed sense. Lakoff claims that “[w]ithin a theory that contains basic-level concepts and image schemas, it is still possible to have rules of semantic composition that form more complex concepts from less complex ones.”¹⁸⁷ The sketch of CMT provided above illustrates that one aim of the theory is economically explaining systematic conceptual structure by means of various cognitive mechanisms, including mappings like conceptual metaphor and conceptual blends. Though Fodor would certainly disagree, there are good reasons to think that CMT satisfies the relaxed compositionality desideratum.

Only the content desiderata remain. CMT seems well positioned to satisfy the cognitive-content requirement (recall that this desideratum requires a theory of concepts to account for the psychological and referential aspects of concepts that can not be explained in terms of intentional content alone). The nature of embodiment means that concepts arise from our

¹⁸⁴Prinz 12.

¹⁸⁵Prinz 291.

¹⁸⁶Prinz 292.

¹⁸⁷*Women, Fire, and Dangerous Things* 280.

experiences, not as direct representations of the world. Experiences of the same object or event can vary dramatically from person to person, and thus different concepts may refer to the same thing; thus, canines may be “lovely puppies” for one person but “hell hounds” for another. Indeed, the kind of “seeing-as” that occurs in conceptual metaphor — when one conceptualizes love as a journey or a force or a work of art, for example — lends itself to explaining the observed differences in our conceptualizations that motivate the cognitive content desideratum. Explaining how CMT satisfies the desideratum of intentional content is more difficult, as is true for many non-traditional theories of concepts. Prinz argues that the intentional contents of many concepts are natural kinds, collections with boundaries independently determined by nature.¹⁸⁸ His explanation of how concepts attain their intentional contents involves a combination of nomological covariance (concepts involve detection mechanisms, which may be fooled by counterfeits) and etiology (incipient causes and histories of concepts are a contributing factor in reference, helping eliminate problems coming from false positives).¹⁸⁹ It seems that CMT will not be able to co-opt Prinz’s arguments verbatim, as Lakoff has repeatedly denied the existence of natural kinds.¹⁹⁰ The relevant question then becomes, what are the intentional contents of concepts according to CMT?¹⁹¹

Lakoff’s anti-objectivism and its associated rejection of natural kinds does not constitute a wholesale abandonment of realism, as some may be inclined to believe. CMT and the associated philosophical position known as embodied realism represent an attempt to find an empirically responsible middle ground between the unpalatable extremes of objectivism and radical relativism.¹⁹² CMT is not an objectivist position because it holds that it is impossible to eliminate the contribution of our embodied perspective from our understanding. Our categories are part of our experience, not part of a human-independent objective reality.¹⁹³

¹⁸⁸Prinz 4.

¹⁸⁹See Prinz (2002), chapter 9.

¹⁹⁰See, for example, *Women, Fire, and Dangerous Things* page 9, and *Philosophy in the Flesh* page 101.

¹⁹¹Determining the nature of the connexion between thoughts or concepts and the world is a perennial problem in philosophy; obviously it is not possible to definitively resolve it here.

¹⁹²Lakoff uses “objectivism” somewhat idiosyncratically to refer to a kind of unsophisticated realist thinking: there exists an objective reality, our perceptions of that reality are direct but flawed reflections of it, and if we could but overcome those flaws (through precise language and rigorous scientific methodology, for example), we could possess absolute, unmediated, and definitive knowledge of the objective world (*Metaphors We Live By* 186–7). Throughout this dissertation, “objectivism” should be interpreted in this way (and not be taken to refer to Randian philosophy, for example).

¹⁹³Lakoff and Johnson, *Philosophy in the Flesh* 19.

On the other hand, CMT also rejects radical “anything goes” relativism. Humans have some freedom in their conceptualizing, but “there are also a great many conceptual universals.”¹⁹⁴ Thus, in CMT, categories are not natural kinds insofar as they are not mental copies of clusters of properties inherent in the world, yet they are constrained, structured, and granted stability to varying degrees by a world that is independent of humans via embodied interactions with that world.¹⁹⁵ Though our categories are models in us rather than inherent collections in the world, many of them arise unconsciously and automatically in all human beings — albeit, by a more complicated mechanism than direct representational projection — and could therefore be called natural. Moreover, it is possible to make categorizational mistakes when dealing with these natural categories: models may be misapplied! Naturally arising intersubjective consensus seems to be a suitable substitute for full-fledged, objective natural kinds.¹⁹⁶ The unconscious ease with which basic-level categories arise early on in our conceptual development may be responsible for the idea they are objective natural kinds.¹⁹⁷ The following passage from Lakoff and Johnson nicely summarizes their position on natural kinds: “One can believe that objectivist models can have a function — even an important function — in the human sciences without adopting the objectivist premise that there is an objectivist model that completely and accurately fits the world as it really is.”¹⁹⁸

It thus seems that though CMT denies the existence of natural kinds it posits a plausibly suitable equivalent in the form of basic-level concepts. However, this is clearly not the entire story. Many of our concepts do not refer to natural kinds insofar as the things they refer to do not exist in the world: GRIFFONS, for example. In chapter 4, we will see that some people

¹⁹⁴Johnson and Lakoff, “Why cognitive linguistics requires embodied realism” 252.

¹⁹⁵Lakoff and Johnson, *Philosophy in the Flesh* 90. It is worth noting that a more sophisticated understanding of natural kinds that deemphasizes independence from human cognition may be compatible with CMT. To the extent this is possible, parts of Prinz’s arguments may be used to defend CMT. However, it is unclear whether such an understanding of natural kinds would be acceptable to Prinz, who claims to endorse “a strong form of realism,” with natural-kind concepts referring to naturally delineated categories (Prinz 5).

¹⁹⁶Above, it was suggested that Prinz might be an objectivist about natural kinds. Elsewhere, however, Prinz suggests that concepts actually have two kinds of intentional content. The *real content* of a concept is an independent thing in the world and the *nominal content* is how we take a thing to be (Prinz 277). It seems that this picture could be made compatible with CMT with a little tweaking. In particular, this would involve deemphasizing the idea of “real essences,” though it seems Prinz would not be happy about this (Prinz 282).

¹⁹⁷Lakoff, *Women, Fire, and Dangerous Things* 31–8.

¹⁹⁸*Metaphors We Live By* 219.

believe that mathematical concepts fall in this class. While traditional theories positing concepts to be reflections of structures inherent in the world may have difficulties handling such vacuous concepts, CMT has no such difficulty.¹⁹⁹ By understanding the intentional content of concepts to be world-grounded interactive experiential models rather than worldly things in themselves, CMT causes controversy by defying tradition but simultaneously situates itself in a good position for explaining vacuous concepts and other puzzle cases.

CMT stands up quite well as a theory of concepts insofar as it seems able to satisfy Prinz's desiderata. The above brief discussion shows that it excels at satisfying scope and acquisition, while further details may be desired in the case of intentional content and the related desiderata of publicity and categorization. Such details may already exist somewhere in the expansive Lakovian corpus or they may need to be newly created; the above discussion has shown that, if the latter is the case, a promising tactic may be to adapt Prinz's approach so that it fits with CMT. Gibbs' claim that concepts should be viewed "not as fixed, static structures but as temporary representations that are dynamic and context-dependent"²⁰⁰ also recommends the creation of a Lakoff-Prinz hybrid theory. Showing that CMT satisfies — or at least has the potential to satisfy — Prinz's desiderata provides an initial defense against a range of objections and criticisms. This positive approach is preferable to considering specific criticisms of the Lakovian theory of concepts because of its efficiency. Indeed, many of the criticisms in the literature involve misinterpretations of CMT that are best defended against by removing the ambiguity or lack of understanding that facilitates them. One example of such a criticism is Marina Rakova's claim that Lakoff's embodied realism is problematically empiricist about concept acquisition, rejecting all traces of conceptual nativism; this would make it difficult — if not impossible — for embodied realism to account for a variety of empirical results.²⁰¹ Explaining how CMT satisfies Prinz's desiderata shows that this criticism is misguided insofar as it overlooks the central claim that all experiences involve interactions between the environment and embodied perceptual and cognitive capacities that are

¹⁹⁹Prinz brings this up as an important objection, especially against etiological theories of intentional content (239).

²⁰⁰Raymond W. Gibbs, "Why many concepts are metaphorical," *Cognition* 61 (1996): 313.

²⁰¹Martina Rakova, "The philosophy of embodied realism: A high price to pay?" *Cognitive Linguistics* 13.3 (2002): 219.

not derived from experience; embodied realism is demonstrably not extremely empiricist and antinativist as Rakova claims.²⁰² I will conclude this chapter by considering a few specific objections against Lakoff and make some suggestions about how they may be overcome.

One objection that has been raised by several authors is that Lakoff often argues against straw men. Leezenberg presents this as a continuation of the criticism that Lakoff rarely argues against specific authors, claiming that “[t]he ‘objectivist tradition’ they fulminate against is not ‘fundamentally misguided’ or ‘humanly irrelevant’ but simply nonexistent.”²⁰³ At times, Lakoff’s antiobjectivist arguments exhibit a seemingly hyperbolic character. However, Lakoff is not always so grandiose:

the myth of objectivism is not itself objectively true. But this does not make it something to be scorned or ridiculed. The myth of objectivism is part of the everyday functioning of every member of this culture. It needs to be examined and understood. We also think it needs to be supplemented — not by its opposite, the myth of subjectivism, but by a new experientialist myth, which we think better fits the realities of our experience.”²⁰⁴

Elsewhere, Lakoff explicitly acknowledges that his arguments are aimed at a general, idealized paradigm rather than the nuanced views of specific individuals.²⁰⁵ While Leezenberg is almost certainly correct that no contemporary philosophers adopt a hard line, capital-T Truth objectivist stance, Lakoff also seems to be correct in claiming that various objectivist doctrines are still pervasive, both in our everyday thinking and in our academic pursuits. It is important to note that the straw man objection is one of several criticisms focusing on Lakoff’s arguments against objectivism, but, apart from playing a motivational role, these arguments are independent from the constructive theorizing behind CMT and embodied realism.²⁰⁶ Therefore, whether one believes that the straw man objection is successful or not has little bearing on the positive claims of CMT which are the subject at hand.

Another criticism leveled against CMT is that many of the key notions involved in the the-

²⁰²Johnson and Lakoff, “Why cognitive linguistics requires embodied realism” 247–51.

²⁰³Leezenberg 137. Murphy expresses a similar view, though perhaps somewhat more charitably than Leezenberg: “This monolithic view [that Lakoff and Johnson call objectivism] is one that many psychologists would not want to commit themselves to” (“On metaphoric representation” 179).

²⁰⁴Lakoff and Johnson, *Metaphors We Live By* 186.

²⁰⁵*Women, Fire, and Dangerous Things* 157.

²⁰⁶Leezenberg 137.

ory (culture, meaning, and imagination, for example) are carelessly defined.²⁰⁷ If one accepts the clichéd aphorism “the first rule of philosophy is ‘define your terms’ ” this alleged carelessness would seem to be an egregious oversight. In making this criticism, Leezenberg seems to erroneously assume that CMT is exclusively philosophical rather than an interdisciplinary theory based upon empirical evidence. While this observation may weaken the criticism, it does not altogether defeat it. However, there is reason to believe that this is objection is not devastating. The above tenet is strongly tied to the classical theory of concepts that is incompatible with CMT’s observation that much conceptual structure is non-definitional. In particular, the terms singled out by Leezenberg are examples of abstract concepts with structure that defies straightforward definition. Despite attracting a certain amount of criticism, the fact that CMT not only allows but sometimes requires a certain looseness in interpretation of terms seems to be one of the strengths of the theory, as it allows for a certain amount of compatibility with what might appear to be competing theories.²⁰⁸ Correspondingly, as the previous paragraph notes, some of the most objectionable passages in the CMT corpus occur when Lakoff makes overly definitive claims that conflict with this flexibility. However, many pages have been written since CMT’s inception with the aim of gently clarifying key notions without becoming contradictorily exact; this work is often a frustratingly slow and lengthy process, but has been relatively successful in providing a clearer understanding than was available when it started. While further careful explanation of the core concepts in CMT is certainly desirable, careless attempts to provide exact definitions of the terms involved might contradict the Lakovian theory of concepts.

A range of criticisms focus on specific conceptual metaphors discussed in the Lakovian corpus. One such criticism holds that certain conceptual metaphors are worryingly arbitrary, since alternative, related conceptual metaphors exist that could perform the same explanatory role equally well. For example, Jackendoff and Aaron question why *LIFE IS A FIRE* should have priority over *LIFE IS SOMETHING THAT GIVES OFF HEAT* or *LIFE IS A FLAME*.²⁰⁹ CMT theorists would likely respond that the priority in this case derives from the fact that *FIRE* is a basic-level concept, whereas the other two source domains are more general and more specific

²⁰⁷See, for example, Leezenberg, page 138 and Black, “Review” page 209.

²⁰⁸This property of CMT is put to good use in chapter 4.

²⁰⁹Jackendoff and Aaron 324.

respectively. Additionally, Lakoff and Johnson stress that the notations they use to refer to conceptual metaphors are names for, rather than accurate descriptions of, the underlying mappings.²¹⁰ Thus, it is possible that the three different metaphor names above refer to the same underlying mapping. Another type of criticism claims that some specific attribution of linguistic utterances as symptoms of an underlying conceptual metaphor is post-hoc and erroneous, either because a different conceptual metaphor is actually responsible or because the mapping named does not actually exist or is not metaphorical in nature.²¹¹ Max Black, for example, claims that *TIME IS MONEY* is not a metaphor but rather the sentence “Time is money” is merely an adage.²¹² Finding some conceptual metaphor to be implausible or even outright incorrect does not constitute an objection to CMT but only one piece of potential evidence counting against it. Even if the details of *all* of the specific conceptual metaphors presented in the Lakovian corpus turn out to be incorrect, this does not constitute a definitive refutation of CMT; that is, a failure to correctly describe any existing conceptual metaphors does not rule out the possibility that such mappings do actually exist. Moreover, while the names of Lakoff’s metaphorical mappings can make them seem spurious or optimistically simplistic, some of them have been the subject of extensive research that has provided a body of evidence supporting their existence; examples include Michael Reddy’s *CONDUIT* metaphor and Lena Boroditsky’s work on time metaphors.²¹³ Thus, while these objections to specific conceptual metaphors should be considered and addressed by CMT researchers, they are not fatal to the theory.

A related objection, first noted by Black, contends that conceptual metaphor analysis contains problematic circularity.²¹⁴ Murphy states the problem in the following way: “linguistic data are used to identify metaphors, but the main concrete predictions the theory makes are about similar linguistic and psycholinguistic data.”²¹⁵ McGlone expresses the same concern, advising conceptual metaphor theorists to “abandon circular reasoning... and seek

²¹⁰ *Philosophy in the Flesh* 58.

²¹¹ Jackendoff and Aaron 330–1.

²¹² “Review” 210

²¹³ See Reddy (1979) and Boroditsky (2000) for more details.

²¹⁴ Black, “Review” 209.

²¹⁵ Murphy, “On metaphoric representation” 200.

substantiation of their claims that is independent from the linguistic evidence.”²¹⁶ Regardless of how plausible or elegant a theoretical posit may be, empirical legitimacy requires that it make testable predictions about phenomena sufficiently distinct from those that motivate it. Murphy suggests that CMT can be strengthened in this respect by providing “predictions about memory, problem solving, induction, measures of conceptual structure (such as typicality and categorization), learning and performance.”²¹⁷ While CMT may have been vulnerable to this objection in its origins, more recent work has worked towards overcoming this objection. Lakoff and Johnson note that there are at least nine types of empirical evidence in support of CMT and, while most of these are linguistic in nature, they involve notably different facets of language.²¹⁸ One of the nine types, spontaneous gesture study, is of particular interest because it provides a nonlinguistic source of evidence; experiments show that the unconscious gestures people make while speaking are frequently consistent with the conceptual metaphors posited to be behind the utterances synchronous with the gestures. For example, while talking about balancing a variety of viewpoints, one subject was observed to hold their hands out with palms cupped upwards, and move them alternately up and down, gesturally mimicking a set of scales; in this case, both the utterance and the gesture are interpreted as realizations of the conceptual metaphor *IMPORTANCE IS WEIGHT*.²¹⁹ This observed multimodality supports the idea that Lakovian metaphors reach deeper than language and structure our concepts. Gallese and Lakoff’s 2005 paper makes some important initial steps towards grounding CMT in neurobiological evidence, and includes a discussion of some future experiments that could help empirically validate their theory.²²⁰ Though the nonlinguistic evidence in favour of CMT may be too scant at present for some critics, recent developments suggest that the situation may improve in coming years.

Another objection to CMT is that it “enlarges the scope of the term ‘metaphor’ well beyond the standard use of the term.”²²¹ While a cursory perusal of Lakoff’s work may give the

²¹⁶McGlone 115.

²¹⁷Murphy, “On metaphoric representation” 200.

²¹⁸“Why cognitive linguistics requires embodied realism” 250.

²¹⁹Müller 96.

²²⁰Vittorio Gallese and George Lakoff, “The Brain’s Concepts,” *Cognitive Neuropsychology* 22.3 (2005): 471.

²²¹Jackendoff and Aaron 325.

impression that all thought is metaphorical, a more thorough and careful reading shows this to be false. Metaphor is extremely pervasive according to CMT but it stops short of being ubiquitous: the term is not vacuous but picks out a useful distinction. Concepts have varying amounts of implicit, non-metaphorical structure, and can be said to be “literal” to the extent they are comprehended or utilized without any use of conceptual metaphor. Some primitive concepts are entirely non-metaphorical, while even highly abstract concepts like LOVE have a scant literal skeleton.²²² Sentences are non-metaphorical when their interpretation requires only literal facets of concepts; Lakoff says the sentence “the cat is on the mat” is not metaphorical.²²³ However, a new problem arises: how does one decide whether a particular instance of language should be interpreted as metaphorical? Leezenberg states the problem in the following way: “[Lakoff and Johnson] reject the ‘objectivist’ accounts that treat metaphor as semantically deviant or arising from a defective ‘literal meaning,’ but offer no clear alternative, even though it is obviously necessary. After all, some sentences can be interpreted either literally or metaphorically, and the same sentence may receive different metaphorical interpretations in different contexts.”²²⁴ CMT has several avenues of response available. First, recall that CMT claims that the majority of conceptual metaphor use is automatic and unconscious; therefore, the production and interpretation of metaphorical sentences in CMT do not require conscious recognition of metaphoricity the way other theories do and, to the extent this is true, the objection simply does not apply to CMT. However, given that we do consciously recognize sentences as literal or metaphorical on occasion, and given that the interpretation of metaphorical language sometimes requires conscious effort, something should be said about linguistic metaphor detection in CMT. Contrary to Leezenberg’s claim, Lakoff comments that there are occasionally cases where a metaphorical interpretation is algorithmically derived from the literal meaning; unfortunately, he does not elaborate and we are thus forced to speculate what he might have meant by this.²²⁵ One mechanism by

²²²While Lakoff tells us “there is...an extensive range of non-metaphorical concepts” (“Contemporary Theory of Metaphor” 205), he provides no specific examples. The most basic concepts, such as UP, seem to be the likely candidates, but even such concepts with significant amounts of direct experiential structure have the potential to receive additional metaphorical structure.

²²³Lakoff, “Contemporary Theory of Metaphor” 205.

²²⁴Leezenberg 139. Similar charges are levied by Mac Cormac (1985), Jackendoff and Aaron (1991), and McGlone (2007).

²²⁵“Contemporary Theory of Metaphor” 205.

which an audience may come to interpret a sentence metaphorically is if the author of the sentence explicitly directs them to do so (“She has a monkey on her back, metaphorically speaking”). An author may also direct their audience to a metaphorical interpretation implicitly through phonological or graphological emphasis, associated gestures, or particular sentence construction (including, but not limited to, word choice and word order).²²⁶ For example, the combination of adjective choice and emphasis in the sentence “If my *flighty* niece is late again, I shall have to clip the wings of the butterfly” suggests an interpretation utilizing the metaphor MY NIECE IS A BUTTERFLY.²²⁷ Though Lakoff consistently rejects the traditional literal/metaphorical dichotomy, in an oft-overlooked paper he contends that many of the problems with that distinction arise because the standard understanding of literality conflates four distinct and non-convergent senses in which an instance of language can be literal:

1. **Conventional literality** occurs when language is used in an unembellished, straightforward, and direct way.
2. **Subject-matter literality** involves talking about some domain of subject matter in the ordinary way.
3. **Nonmetaphorical literality** occurs when an instance of language is directly meaningful and interpreted without the use of any conceptual metaphors.
4. **Truth-conditional literality** is exhibited by utterances which are capable of being objectively true or false.²²⁸

Of these senses, only nonmetaphorical literality contrasts with linguistic metaphoricity.²²⁹ The opposition between nonmetaphorical literality and metaphoricity is precisely the distinction invoked at the beginning of this paragraph; it is possible that more details of the conscious, contextual application of this distinction could be obtained. Also, though Lakoff

²²⁶Support for this idea can be found in Lakoff’s work: “Grammar can also play a role in activating a metaphor” (“The Neural Theory of Metaphor” 35). See the quotation in the next paragraph for more specifics about how Lakoff currently thinks metaphorical language works.

²²⁷This metaphorical sentence is modified from an example of Martinich (“A Theory for Metaphor” 448).

²²⁸George Lakoff, “The Meanings of Literal,” *Metaphor and Symbolic Activity* 1.4 (1986): 292.

²²⁹Lakoff, “The Meanings of Literal” 295.

argues that the four senses of literality are non-convergent, this does not entail that they are mutually exclusive; if one or more of the other senses of literality fails to obtain for some sentence, this may sometimes suggest that exploring metaphorical interpretations is appropriate. Thus, there are several promising avenues that CMT could follow in trying to answer the interpretation question.

Many of the specific criticisms presented thus far are connected to the general issue of how concepts and language are related. That these problems require individual attention is partially a consequence of Prinz's omission of language requirements from the list of desiderata used to evaluate CMT in this chapter. The sketch of CMT provided above is consistent with what Gibbs calls the *cognitive wager*, a commitment to the assumption that linguistic structures have their basis in more general conceptual and experiential features of cognition; this commitment places the burden of proof with those who make the contrary assertion that language is autonomous from other modes of thought.²³⁰ CMT portrays sentences as mere symptoms of underlying cognitive structures and mechanisms; thus, there seems to be little relevant difference between a Shakespearean sonnet and the semi-articulate ramblings of a simpleton foreigner insofar as both pieces of language invoke the same conceptual metaphor, for example. Lakoff's more recent, neurology-oriented writings seem consistent with this picture:

The neural theory of language allows us to understand better why language is so powerful. Let's start with words. Every word is defined via linking circuit to an element of a frame — a semantic role. Because every frame is structured by a gestalt circuit, the activation of that frame element results in the activation of the entire frame. Now, the frame will most likely contain one or more image-schemas, a scenario containing other frames, a presupposition containing other frames, may fit into and activate a system of other frames, and each of these frames may be structured by conceptual metaphors. All of those structures could be activated simply by the activation of that one frame element that defines the meaning of the given word. In addition, the lexical frame may be in the source domain of a metaphor. In that case, the word could also activate that metaphor. In the right context, all of these activated structures can result in inferences.²³¹

This description of the connexion between words and neural concepts is relatively detailed compared to other related passages from earlier in the CMT corpus; however, it is consistent

²³⁰Raymond W. Gibbs, *The Poetics of Mind* (New York: Cambridge UP, 1994), 15.

²³¹Lakoff, "The Neural Theory of Metaphor" 34.

with the earlier theory in that its emphasis on the conceptual contributes to an underemphasis on the specifics of the linguistic side of the relationship, as Cornelia Müller here describes:

Remember that the ultimate goal of Lakoff and Johnson's enterprise is, at its core, an experiential theory of human understanding that is based on processes of figurative thinking... language is primarily of interest insofar as it provides insights into the functioning of the mind and more specifically into general cognitive mechanisms. These are then projected back to explain verbal metaphorical expressions in terms of conceptual metaphors. This procedure, important as it is, necessarily ignores linguistic variation. Its focus is on the common principles that underlie language and language use and not on variation, be it caused by differences in interindividual usage or in the conditions under which language is used... [t]he assumed conceptual metaphors may well be plausible on the level of a language community and probably also on the level of a collective mind. Yet the extent to which they guide the understanding and actions of individuals is a fundamentally different question.²³²

Authors like Gibbs and Müller, though generally sympathetic to the Lakovian position, criticize CMT for this lack of balance and its dearth of details in describing the relationship between the conceptual and the linguistic, and use these shortcomings as motivation in their own research. I will conclude this section on the problems facing CMT by briefly discussing two promising avenues that may help overcome this final criticism.

In *Metaphors Dead and Alive, Sleeping and Waking*, Müller claims that “the distinction between dead and live metaphors is indeed at the core of traditional as well as of cognitive metaphor theories [but]... it has not received the profound attention of metaphor scholars, resulting in the lack of its systematic integration into theories of metaphor.”²³³ One of the foundational elements of her theory is a “distinction that metaphor theories tend not to reflect upon systematically: the distinction between the collective level of linguistic and/or conceptual systems and the individual level of representation and actualization of those systems.”²³⁴ In speaking about metaphors at the “level of the system,” Müller is in part referring to words and phrases as they might feature in a lexicon or a phrasebook, relatively fixed in meaning or interpretation (at a given point in time) and detached from specific authors, audiences, and situations. However, it seems that “system” must refer to something more than this

²³²Müller 47.

²³³Müller 10.

²³⁴Müller 12–3.

given that she claims metaphors may also be instantiated in non-linguistic modalities such as picture and gesture. Hence, Lakovian conceptual metaphors would seem to constitute paradigm cases of metaphor at the level of a system: conceptual metaphors are modality-independent, intersubjective (or at least potentially intersubjective), and “subject to general cognitive principles.”²³⁵ Whether the system involved is considered to be strictly linguistic or more broadly cognitive or conceptual, Müller claims that for most theories of metaphor, the “relation between the level of the system and the level of individual use is conceived of as relatively unproblematic,”²³⁶ with particular instances of metaphor in discourse oftentimes being regarded as merely derivative or epiphenomenal. Due in part to this frequent bias toward the systematic in the literature, Müller decides to focus her attention on the level of use in her book; however, she is careful to note that “[t]his is not to say that the perspective of individual use should replace the systems perspective or that with regard to metaphors it should replace semantics with pragmatics. Rather, individual use must be considered as a noteworthy dimension of language with its coherent structures and organizational principles and not just as a defective instantiation of whatever system.”²³⁷ That is, by focusing her attention on the level of use, Müller is attempting to bridge the gap between system and use, not to suggest the systems level is eliminable or otherwise unworthy of consideration.²³⁸ She describes her position as follows:

this view complements the concept of conceptual metaphor with a level of verbal metaphor and complements the concept of metaphor as a uniquely verbal phenomenon with a level of metaphor that is subject to general cognitive principles. Hence, it provides support for the general claims of conceptual metaphor theory, while at the same time pointing out the necessity of postulating a linguistic level of metaphoric structure.”²³⁹

Müller’s theory shows promise in several ways. It is a fundamentally dynamic view of metaphor with a firm commitment to empirical sensitivity. It addresses a wide sample of traditional and contemporary theories of metaphor with uncommon objectivity, pointing out their deficiencies while attempting to systematically incorporate — or at least be compatible

²³⁵Müller 16.

²³⁶Müller 13.

²³⁷Müller 14.

²³⁸Müller 15.

²³⁹Müller 16.

with — their various strengths rather than rejecting them wholesale. The resulting synthetic amalgam incorporates multiple dimensions (system versus use, language versus thought) and multiple realms (conceptual, verbal, verbo-gestural, verbo-pictorial) of metaphor, and yields a satisfyingly non-reductive, complicated, and balanced picture of metaphor as possessing multiple levels of organizational structure.²⁴⁰ Though it is neither feasible nor necessary to discuss the details of her theory here, it warrants further consideration by those concerned with developing a unified theory of language, concept, and metaphor. In particular, the idea of forming a hybrid of CMT and Müller’s theory warrants further attention.

Another promising approach would involve attempting to integrate semiotic insights with CMT. Semiotics is the theory of signs, and their interpretation and communicative use. Because signification permeates human existence, semiotics is a loosely unified discipline with broad application. One traditional understanding of semiotics comes from C.S. Peirce: “[Semiosis is] an action, or influence, which is, or involves, an operation of *three* subjects, such as a sign, its object, and its interpretant, this tri-relative influence not being in any way resolvable into an action between pairs.”²⁴¹ This definition shows that language (signs), concepts (interpretants), and the connexion between them are clearly within the purview of semiotics, though they do not exhaust its scope. In semiotic terms, a metaphor can be seen as resulting when the sign of one act of semiosis becomes the object of a secondary semiosis. While this understanding may seem overly simplistic at first, unpacking its implications is a daunting task that is best performed by semioticians; Umberto Eco has done some important work on this problem.²⁴² I will not go into the details of this account here. There are several reasons why I think that attempting to integrate semiotics and CMT could be productive. CMT has been criticized for overemphasizing the conceptual; making it compatible with the semiotic trinity could create a stronger, more balanced theory. On the other hand, unlike purely linguistic theories of metaphor, any semiotic account must involve discussion of interpretants, and is therefore likely to agree with conceptual theories in at least some respects. It is plausible that Müller’s work could be compatible with an explicitly semiotic approach and that the two could work in concert to improve CMT. The introduction of semiotic elements

²⁴⁰Müller 112.

²⁴¹Qtd. in Eco, *Semiotics and the Philosophy of Language* 1.

²⁴²See Eco, *Semiotics and the Philosophy of Language*, chapter 3.

could also fruitfully rejuvenate CMT's relationships with some of its theoretical ancestors. The irreducible trinity of Peirce's definition is often represented pictorially as the triangle of reference, popularized by Ogden and Richards in *The Meaning of Meaning*.²⁴³ As Richards is often given credit for Lakoff's core precept that "metaphor is primarily a matter of thought," the establishment of further consistencies between their work would be a positive development. Going further into history, John Kirby claims that "a semiotic presentation of the Aristotelian model can be made congenial to Lakoff's cognitive approach."²⁴⁴ From the perspective of this document, the most significant advantage of combining CMT and semiotics would be that it would facilitate a productive interaction between Lakoff and Núñez's embodied theory of mathematics and Brian Rotman's semiotic theory of mathematics. Though I am optimistic that the suggestion of reconciling CMT and semiotics could help overcome many of the problems facing the Lakovian theory, I am aware that such an undertaking would take significant effort and patience, and may even be outright impossible. Mercifully, there is no need for me to attempt this; an airtight defense of CMT is not required here.

In chapter 2, a historical sample of purely linguistic theories of metaphor was analyzed and found wanting in various ways. In hopes of bypassing these problematic shortcomings, chapter 3 considered the idea that metaphor could be conceptual in nature. Rather than survey a variety of positions as before, I focused my attention on the foremost theory of conceptual metaphor: Lakoff's CMT. A synopsis of this theory distilled from the voluminous corpus explained how the stable commonalities of human biological embodiment and the environment we occupy combine with the capacity for creating mental metaphorical mappings to yield abstract concepts, successful communication grounded in an intersubjective language, and other facets of human experience. Prinz's desiderata show CMT to be a promising theory of concepts; while it has a few weaknesses, they seem non-fatal and no worse than those observed in other leading theories of concepts. More-telling criticisms target the overemphasis of CMT on concepts, and the corresponding scarcity of details regarding language use and the relationship between the linguistic and the conceptual. In particular, some critics object that CMT does not adequately explain under what circumstances a specific linguistic utterance

²⁴³C.K. Ogden and I.A. Richards, *The Meaning of Meaning* (New York: Harcourt, Brace, and Co., 1923), 11.

²⁴⁴Kirby 538.

receives a metaphorical interpretation. Observations such as this lend credence to claims that CMT is not a sufficient theory of metaphor because not every example of metaphoricity can be adequately construed as a symptom of conceptual metaphor.²⁴⁵ To overcome such criticisms, I suggest that CMT might look to Prinz’s theory of concepts as proxytypes, Müller’s theory of metaphor, or semiotics for inspiration. Indeed, many of the authors criticizing CMT for not giving a complete account of metaphor concede that the insights and evidence of the Lakovian position are at least somewhat meritorious, and incorporate some notion of conceptual metaphor into their own theories.²⁴⁶ Thus, I have not provided an airtight defense of CMT and I have conceded the possibility that better theories of metaphor may exist. Has this entire chapter then been a pointless exercise?

As you undoubtedly surmise, the answer to this question is ‘no.’ While I believe that pursuing a comprehensive philosophical account of metaphor is a worthy goal, it is not the principal aim of this chapter. The primary goal of the first half of the dissertation was to provide a summary of the philosophy of metaphor tailored to the task of examining possible relationships between metaphor and mathematics, to be undertaken in the second half. Chapter 3 focused on assessing whether metaphor can be understood as conceptual, a matter of thought rather than simply a matter of language. An affirmative result has been obtained: while the stronger claim of CMT that *every* metaphor has a conceptual basis has not been adequately verified, there are good reasons to believe that at least *some* conceptual metaphors exist. It is adequate for the purposes of the following chapters that the reader concede that some conceptual metaphors may exist and constitutively structure abstract concepts, including those of mathematics, a relatively weak requirement. Given the formal nature of mathematical language, it seems likely that if metaphors play a role in mathematics, they are more likely to be conceptual than linguistic in nature.²⁴⁷ I have focused discussion in this chapter exclusively on CMT despite the existence of strong competing theories of conceptual

²⁴⁵See, for example, Jackendoff and Aaron: “We are...not convinced that the notion of [conceptual metaphor] characterizes a unified cognitive phenomenon. In fact, having drained from the term ‘metaphor’ much of its traditional content, [Lakoff and Turner] have created a theoretical construct so broad and unstructured that the term ‘metaphor’ may no longer be appropriate” (331).

²⁴⁶For example, see Müller: “the fundamental claims of Lakoff and Johnson’s conceptual metaphor theory now appear to be empirically validated” (51).

²⁴⁷Or, if linguistic metaphors do play a role, they are likely to be very different from the poetic metaphors put forward as paradigm cases by most theorists.

metaphor, including those of Müller, Leezenberg, Jackendoff, Gentner, and Fauconnier and Turner. Note that nearly any theory that posits the existence of conceptual metaphors ought to meet the weak requirement for this chapter mentioned above, so the exact details of how conceptual metaphor is understood are not particularly important; thus, a survey of competing views is unnecessary.²⁴⁸ Given the lack of consensus in the field, I opted to focus on CMT because of its various strengths: its extensive, systematic corpus; its empirical sensitivity; its relatively clear presentation; its emphasis on human biology; and, significantly, the fact that Lakoff has published extensive arguments that mathematics is founded on conceptual metaphor.

²⁴⁸If definitive evidence for one of the competing theories arose, I am confident that the position taken in this dissertation could be modified to accommodate the new results.

CHAPTER 4

MATHEMATICS IS METAPHORICAL

La Mathématique est l'art de donner le même nom à des choses différentes.

(Mathematics is the art of giving the same name to different things).¹

— Henri Poincaré

In this chapter, I argue that mathematics constitutively involves metaphor, that it is philosophically fruitful to conceive of mathematics as metaphorical in some important sense. Though this idea is not a new one — various authors have espoused theories of mathematics in this vein — it remains relatively unknown in both popular and academic circles and is controversial among philosophers and mathematicians alike. This chapter defends the Lakovian theory of embodied mathematics, in part by establishing coherences with other theories of mathematics.

It is clear that the claim that mathematics involves metaphor depends heavily on how these key terms are understood. The main purpose of chapters 2 and 3 was to paint a picture of metaphor as a more complex, diverse, and legitimate phenomenon than the traditional view would have us believe. An acknowledgment that metaphors may sometimes be more than eliminable obfuscating ornamentation is necessary if the above claim about mathematics is not to be rejected as trivial and uninteresting, or outright preposterous. In particular, cautious acceptance of the idea that some metaphors could be conceptual rather than merely linguistic provides substance to the claim, insofar as mathematics is a highly conceptual discipline with a frequent emphasis on precise formal language. Adopting a broad and inclusionary understanding of metaphor as chapter 3 suggests is desirable as it allows for an interesting range of possibilities.

¹Henri Poincaré, “The Future of Mathematics,” *The Monist* 20.1 (1910): 83.

What is mathematics? While this question may be easier to answer than the corresponding one about metaphor, it is not entirely straightforward and requires some explicit attention. Most people know mathematics as a topic they are forced to study in school, a subject consisting of facts and skills pertaining to numbers and shapes. Some of those who do not find the discipline too onerous or repugnant will go on to become professional mathematicians; others, such as scientists and engineers, will rely heavily on mathematical techniques and concepts in their work. Mathematics is not confined to the classroom and the laboratory, however, but is encountered and used by most people on a daily basis, in their financial dealings, creative projects, and time management, for example. Two important ways of seeing mathematics emerge from this brief sketch. One is mathematics as a subject matter, a collection of theories or a system of knowledge. The other important way of seeing mathematics is as a collection of practices, such as proving and calculating; that is, mathematics is something that one *does*.

The approach adopted in this chapter emphasizes mathematics-as-practice. Traditionally, the philosophy of mathematics has focused on mathematics-as-body-of-knowledge and the associated ontological and epistemological issues. Such approaches often consider mathematical practice to be an unfortunate necessity, merely a clumsy means to an elegant and austere end; this point of view has its roots in Plato. However, several perennial problems plague traditional philosophies of mathematics, leading to a lack of consensus in the discipline and suggesting that a paradigmatically different approach is worthy of consideration. Developments in the biological and social sciences over the last century have replaced a void with an ever-increasing body of evidence about mathematical practices. These results help motivate and shape practice-oriented approaches, insofar as it is desirable to have a theory of mathematics that is compatible with the emerging data. Several practice-oriented theories of mathematics have arisen in recent years, though such alternative positions are still very much a minority. Key examples include Lakoff and Núñez's embodied mathematics, Lakatos's quasi-empiricism, Rotman's semiotic approach, and certain strains of mathematical fictionalism. The goal of this chapter is not to provide a novel philosophy of mathematics, but rather to show how the combination of practice orientedness with a progressive understanding of metaphor provides a framework that avoids some classic problems while allowing

for the reconciliation of the best features of multiple theories, both traditional and contemporary. Such an approach will have implications for the ontology and epistemology of mathematics, though answering the traditional demands of these philosophical disciplines is not a primary objective. The aims of this chapter are primarily descriptive, and do not aim to alter mathematical practices that are successful. However, the frustration and loathing that frequently accompany the learning of mathematics suggest that educational practices may not be as successful as they could be; it will be suggested that embracing a practice- and metaphor-oriented approach could help alter teaching methods for the better.

The itinerary for this chapter is as follows. A brief exposition of the strengths and weaknesses of key traditional philosophical theories of mathematics provides background context for the ensuing discussion. Lakoff and Núñez’s theory of embodied mathematics based upon CMT is offered as a promising alternative that accounts for and integrates many of the positive insights of the traditional theories. The presentation of this viewpoint involves discussion of some recent scientific findings concerning mathematical cognition, as such evidence is foundational to the theory. Like the underlying CMT, the theory of embodied mathematics is controversial and has been criticized by both philosophers and mathematicians. It is shown that many of these criticisms involve a fundamental misunderstanding deriving from the fact that they cross paradigms. The chapter concludes with an examination of some other contemporary theories of mathematics that involve metaphor and/or are practice-oriented. In particular, Yablo’s mathematical figuralism, Rotman’s semiotic account of mathematics, and Lakatos’s quasi-empiricism are discussed. Compatibilities between Lakovian embodied mathematics and these other positions are indicated, suggesting directions for future research and the development of a more robust metaphor-dependent account of mathematics.

4.1 Traditional Theories of Mathematics

The first section of this chapter is a short introduction to twentieth-century philosophy of mathematics.² The primary aim of this section is to provide a motivational context for alter-

²Philosophizing about mathematics goes back at least as far as the Pythagoreans. While it would be ideal to examine the entire history of the topic, such an approach is too extensive an undertaking for this chapter. Fortunately, it will be adequate for most purposes of this chapter to view the philosophy of mathematics as

native practice-oriented approaches by exposing flaws in the leading traditional theories. The goal is not to rigorously and exhaustively demonstrate that no adequate philosophical theory of mathematics currently exists, nor to definitively reject all formulations of the traditional approaches, but only to show that there is no consensus in this controversial discipline and that there are serious difficulties that each of the classical theories must address if they are to be considered tenable. There is a significant literature available to those desiring a more detailed discussion of the controversy between the traditional positions than is given here.³ An additional motivation for considering alternative positions that comes out of this section is that the hypothesis that metaphor plays a constitutive role in mathematics does not cohere well with the traditional theories; this is far from a fatal objection but, for the purposes of this chapter, does provide a reason to consider some less mainstream theories.

The philosophy of mathematics stands at the crossroads of several major areas of philosophy, including metaphysics, epistemology, logic, and the philosophy of language. It emerges as a distinct philosophical subdiscipline because of the unique character of mathematics among the various areas of human knowledge: it seems more rigorous, more abstract, more universal, and more timeless.⁴ Accordingly, the central problems in the philosophy of mathematics consider how and why mathematics has this distinctive character, or, if it does not, why it seems to. Traditionally, attempts to answer these problems involve discussion about the nature of mathematical objects, the nature of mathematical truth, and the relationships between these objects, truths and ourselves.⁵ Philosophical theories of mathematics can be evaluated both on how well they address these key issues, as well as the extent to which they cohere with our general understanding of non-mathematical objects and truth. This general understanding provides a framework for evaluating and comparing theories of mathematics.

Though mathematical platonism has its roots in Plato's theory of Forms, the former has

originating with Frege. However, the reader (and the author) should remain cognizant that, in actuality, the tradition extends much farther into the past and Frege did not invent the subject *ex nihilo*.

³For an exemplary single-volume treatment of these issues, see *The Oxford Handbook of Philosophy of Mathematics and Logic* (2005).

⁴It should also be noted that there is an important historical dimension to this: mathematics was recognized as a distinct area of knowledge relatively early on, and has had a place serving as the preeminent model for knowledge since the origins of philosophy. Indeed, the ancient Greek root of the word "mathematics" is $\mu\acute{\alpha}\theta\eta\mu\alpha$, which is translated as "knowledge" or "something learned" ("mathematic, n. and adj." *OED Online*).

⁵Mac Lane 4.

become largely divorced from the latter. Contemporary mathematical platonism is defined by the core belief that mathematical objects are real, abstract, and exist independent of agents.⁶ Thus, for platonists, mathematical objects — such as numbers, sets, functions, spaces, polyhedra, etc. — only differ from everyday non-mathematical objects in that they are abstract rather than concrete. The primary advantage of platonism is that a distinct account of mathematical truth is unnecessary: mathematical propositions are strongly analogous with everyday propositions and attain their truth values in the same way.⁷ Typically, platonists hold that the truth value of a proposition derives from whether it accurately reflects the objective facts. It is the abstractness of mathematical objects that distinguishes mathematics from empirically based subjects. Whereas the physical world is in flux and its truths are dynamically contingent — the cat was on the mat earlier, but is now hunting birds in the garden — the abstract objects of mathematics are unchanging, outside of space and time, so truths about them inherit an unyielding robustness. Moreover, because mathematical objects are abstract, our knowledge of them does not depend on our physical sensations and experiences, and is therefore *a priori*. Varieties of platonism have been the foremost philosophy of mathematics for over two millennia because they seem to provide a simple yet robust account of mathematics that fits with standard semantics and other traditional ideas about language use, and with the experiences of practicing mathematicians.

But the primary flaw in the platonist position also has to do with the abstractness of mathematical objects. First, the principle of parsimony provides a strong reason to consider theories of mathematics with simpler ontologies; a theory which is able to explain mathematics without recourse to abstract objects will be preferable to platonism. Moreover, the common understanding of the world predominated by the physical sciences comes with a tendency to find platonic mathematical objects worryingly mystical. While we have a causal story explaining how sensory perception of physical objects occurs, there is no parallel explanation of how we come to experience abstract objects; this is one area where physical and abstract objects are not relevantly similar. This is often referred to as the *access problem*

⁶Øystein Linnebo, “Platonism in the Philosophy of Mathematics”, *Stanford Encyclopedia of Philosophy*, 2 May 2012. I will adopt the convention of using the uncapitalized “platonism” as an abbreviation for mathematical theories that possess this core belief.

⁷James Robert Brown, *Philosophy of Mathematics*, 2nd ed. (New York: Routledge, 2008), 12.

(or, sometimes, Benacerraf’s problem): how can physical beings come to know abstract objects that lack causal efficacy?⁸ Plato’s solution to the access problem was to postulate that learning is recollecting knowledge of the Forms gained during the periods between death and rebirth when we, like mathematical objects, are disembodied.⁹ This fantastical explanation is one of the reasons that most contemporary platonists distance themselves from the traditional theory of Forms. However, no alternative solution to the access problem is widely agreed upon.

The rejection of platonism necessarily involves a denial of at least one component of its core conception of mathematical objects: existence, abstractness, or human-independence. One possibility is to reject abstractness but retain the other two components, resulting in an extreme and untenable variety of mathematical empiricism where all objects are known only through sensory perception.¹⁰ There is an obvious objection that empiricism must overcome, which is often presented by platonists as an argument in favour of the abstractness of mathematical objects. It seems impossible that certain mathematical objects could exist in the physical world. For example, if the universe is understood to consist of finitely many discrete particles, that makes it difficult to empirically explain the uncountability of the integers and, worse still, the infinite continuum of real numbers.¹¹ Moreover, humans perceive only three spatial dimensions and one temporal dimension, whereas mathematicians routinely work with spaces and objects with many more than four dimensions; clearly, such mathematical objects cannot exist within the world as we perceive it. Any contemporary empirical account must include an answer to this objection among its foundations.

A more telling objection works against the shared realist dimension of platonism and

⁸Brown 16–7. Brown does not provide a positive account of how we are acquainted with mathematical objects, but instead rejects the access problem on the grounds that Einstein-Podolsky-Rosen “spooky action at a distance” provides a fatal counterexample to the causal theory of knowledge (17–9). In addition to failing to provide a positive account, it is not clear that Brown’s approach does the work he wants it to given that “Benacerraf’s problem is remarkably robust under variation of epistemological theory” (Horsten). See Benacerraf’s “Mathematical Truth” (1973) for the original statement of the objection.

⁹See Plato’s *Meno* and *Phaedrus* for details.

¹⁰Historically, few authors have defended any form of mathematical empiricism (John Stuart Mill is the canonical example), and none of those few has taken this naive hard line, as far as I am aware. However, the contemporary scientific paradigm has given rise to a new interest in more sophisticated varieties of mathematical empiricism. For example, Philip Kitcher describes his approach in *The Nature of Mathematical Knowledge* as empirical (4).

¹¹Brown, *Philosophy of Mathematics* 13–4.

empiricism. Mathematical realists believe that independently existing mathematical objects are necessary to ground objective mathematical truths. Stephen Yablo's concise objection to realism takes the form of a dilemma: either our conception of numbers is determinate or it is not.¹² If our number concept is determinate, then that conception alone is sufficient for the objectivity of truths about numbers whether it actually refers to number objects or not, making the objects redundant. If our number concept is not determinate then an explanation of how it successfully refers to an objectivity-providing structure is required. Yablo contends that this can only be done if only one structure in the extension of our conception exists. However, this scenario admits the possibility of Putnamian Twin Earth cases where a person internally identical to me makes an arithmetical statement that would be true coming from my mouth ("7 is a prime number," for example), but is false on Twin Earth because it refers to twintegers that differ from the integers. Hence, our arithmetical concepts are revealed to be externalist and the characteristic necessity of mathematical statements is compromised, entailments that are generally held to be unacceptable by realist philosophers.¹³ Such general arguments against mathematical realism in concert with the specific difficulties facing platonism and empiricism provide motivation to consider other — and, particularly, antirealist — positions.¹⁴

Two related advents in mathematics in the late nineteenth century allow for the development of serious alternatives to platonism. The first was a bevy of improvements in formal logic, set theory, and axiomatics. The second was the development of non-Euclidean geometries. In particular, the latter provided evidence against the platonistic idea that there is only one true mathematics. However, the former is directly responsible for one of the first alternative approaches in the philosophy of mathematics. Logicism is defined by the belief that mathematics is, in some nontrivial sense, reducible to logic. This usually means

¹²For Yablo, a determinate conception is one for which any statement made about an exemplar of the concept is unambiguously and decidedly true or false ("Go Figure" 194).

¹³Yablo, "Go Figure" 194–5.

¹⁴The taxonomy of mathematical theories is not as clear as the discussion in this section makes it out to be; for example, there may be positions self-identifying as varieties of platonism or empiricism that are less committed to the independence of mathematical objects. Likewise, not all of the positions considered below are necessarily incompatible with realist assumptions. For example, Frege is often identified as both a platonist and a logicist. The reader is once again reminded that this overview of the philosophy of mathematics is drastically simplified and generalized by necessity.

attempting to show that all mathematical truths are tautological, logically derivative from a set of self-evident axioms. Accomplishing this would establish an ontology-independent foundation for mathematical knowledge; for example, “All polytopes are polytopes” is a tautology regardless of whether polytopes exist. The emphasis in logicism is on proof and the relationships between concepts. Gottlob Frege is generally recognized as the first logicist, though he drew inspiration from Leibniz’s conjecture that arithmetic is reducible to logic.¹⁵ The revolutionary logical developments contained in his *Begriffsschrift* allowed Frege to make the first concerted attempt to describe arithmetic as a formal deductive system in his two-volume *Grundgesetze der Arithmetik*, an undertaking that previous logics were ill equipped to handle. However, a major flaw in Frege’s system was exposed by Bertrand Russell just as the second volume of the *Grundgesetze* was going to press, namely that Frege’s Basic Law V entails Russell’s paradox and, therefore, is inconsistent.¹⁶ In order to circumvent the paradox, Russell developed his theory of types, an alternative to naive set theory which restricts problematic self-referentiality. This laid the groundwork for Whitehead and Russell’s *Principia Mathematica*, arguably the pinnacle of logicist achievements and one of the most significant publications of the twentieth century. Though it is less read today, the influence of the *Principia* permeates contemporary mathematics.

While logic is undeniably related to mathematics, logicism fails to be an adequate philosophy of mathematics. Before looking at the criticisms, some positive contributions of logicism should be noted. Logicist ideals have been a part of mathematics since Euclid: every mathematician aspires to be as systematic, clear, and rigorous as the *Elements*. Logicism takes this impulse to its natural conclusion, attempting to expose the entirety of mathematics as a single axiomatic system. In trying to achieve this goal, logicists not only generated a variety of important fundamental mathematical results, but changed the way mathematics was practiced by implementing new techniques and standards for proofs. Improved symbolization and other developments in formal logic also changed the way philosophy was practiced, resolving some long standing problems and giving rise to new questions. The reductive successes achieved by the logicists paved the way for the development of the computer, which utilizes

¹⁵Michael D. Resnik, *Frege and the Philosophy of Mathematics* (Ithaca: Cornell UP, 1980), 13.

¹⁶Resnik 211–2.

electronic logical circuitry to perform arithmetic calculations; questions concerning the nature of mathematics qua logical system (specifically, the *Entscheidungsproblem*) motivated the creation of the theoretical Turing machine.¹⁷ Despite its mathematical and philosophical successes, logicism is untenable as a foundational philosophy of mathematics. First, it is not possible to reduce mathematics to logic without including set theory (or some equivalent); only absolute purists would claim that the logicist enterprise fails because of this.¹⁸ However, even more forgiving scholars acknowledge that mathematics requires certain axioms and inferential rules that are distinctly non-logical, including the axiom of infinity and the principle of mathematical induction.¹⁹ Moreover, there are some mathematical statements — the continuum hypothesis is the canonical example — which can be neither proved nor disproved from the axioms of set theory and which, therefore, must stand alone as independent axioms.²⁰ Another criticism is that logicism takes for granted rather than shows that logic itself is certain and reliable; thus, even if the reductions were successful, it would remain unclear that the foundations of mathematics are secure.²¹ Third, logicism has difficulty explaining the acquisition and development of mathematics, which does not begin with the axioms. While much of mathematics may be generated from a small collection of axioms, nobody learns or performs arithmetical calculations in the manner presented in the *Principia*. Logicism, therefore, does not accommodate a wide range of our mathematical experiences.²² Arguably, the most devastating blow to logicism comes from Gödel's incompleteness results; I shall postpone discussion of this fatal criticism briefly to introduce formalism, a philosophical sibling of logicism equally shaken by that objection.

From some perspectives, the distinction between logicism and formalism seems slight: both viewpoints seek foundational certainty in the conception of mathematics as a formal axiomatic system rather than in objective abstract objects. Where they differ is on their

¹⁷Alan Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem," *Proceedings of the London Mathematical Society*, Series 2, 42 (1937): 230–65.

¹⁸Paul Benacerraf and Hillary Putnam, "Introduction," *Philosophy of Mathematics: Selected Readings*, Eds. Benacerraf and Putnam (Englewood Cliffs: Prentice-Hall, 1964), 10.

¹⁹It is noteworthy that every example of a non-logical mathematical axiom I have encountered involves infinity in some way.

²⁰Paul Ernest, *Social Constructivism as a Philosophy of Mathematics* (Albany: SUNY Press, 1998), 16.

²¹Ernest 17–8.

²²This line of criticism is similar to one presented by Wittgenstein in *Remarks on the Foundations of Mathematics*, Part II (1956).

understanding of mathematical truth. Recall that for the logicist, mathematical truths are logical truths. On the other hand, formalists hold that the formal system of mathematics is purely syntactic, the result of the repeated application of sanctioned symbol-manipulation rules to an ever expanding collection of symbolic strings.²³ These symbolic strings are thus devoid of meaning unless some interpretation is imposed on them.²⁴ For the formalist, then, the key issue is not establishing that mathematics is true in the conventional sense but rather that the formal system is consistent, free from paradox and contradiction. Formalists have an advantage over logicists in that they are free to use non-logical or seemingly ad hoc rules and axioms so long as they are consistent with their system. On the other hand, a common complaint about formalism is that it seems ill equipped to explain why the meaningless game of mathematics should be so “unreasonably effective” (as Wigner so aptly put it), or why mathematics ought to be privileged over any other meaningless game.²⁵ However telling this criticism may be, the foundational ambitions of both logicism and formalism were effectively terminated by one of the most astounding intellectual results of the twentieth century.

In 1900, David Hilbert, one of the foremost mathematicians of his day and noted formalist, presented a collection of 23 significant open problems to the mathematical community. The second of these problems asked for a proof of the consistency of the axioms of arithmetic (that is, that the system derived from those axioms does not contain a contradiction).²⁶ Kurt Gödel’s graduate research aimed to provide such a proof, but instead culminated in the publication of his infamous incompleteness theorems in 1931. These results can be

²³Another way of putting this is that logicists thought mathematics had a null subject matter and “dealt with pure relations among concepts” whereas formalists thought that the subject matter of mathematics was comprised of symbolic expressions (Benacerraf and Putnam 9). Note that no account of metaphor takes it to be purely syntactic, and thus the intuition that metaphor plays a constitutive role in mathematics is incompatible with formalism.

²⁴David Hilbert, the father of formalism, contended that mathematics was composed of two types of formulae: “first, those to which the meaningful communications of finitary statements correspond; and, secondly, other formulas which signify nothing and which are the *ideal structures of our theory*” (“On the Infinite” 146). That is, it seems that Hilbert thought that elementary arithmetical statements involving finite integer quantities can be certain perceptual truths (as Kant did), but that statements involving infinity are meaningless but instrumentally useful extensions of the finitary (Brown 70–1). This provides a reminder that formalism — and, indeed, each of the viewpoints considered — is more complicated than the brief sketch I present here.

²⁵Eugene Wigner, “The Unreasonable Effectiveness of Mathematics in the Natural Sciences,” *Communications in Pure and Applied Mathematics* 13.1 (1960): 1.

²⁶David Hilbert, “Mathematical Problems,” *Bulletin of the American Mathematical Society* 8.10 (1902): 447.

paraphrased non-formally as:

1. Any consistent formal system containing ordinary arithmetic is incomplete (contains a sentence such that both the sentence and its negation are unprovable within the system).
2. It is not possible to prove the consistency of such a formal system within that system.²⁷

It is the second of these two results that deals the fatal blow to projects seeking to establish certain axiomatic foundations for mathematics; as Paul Ernest puts it “[t]he second incompleteness theorem showed that in the desired cases consistency proofs require a meta-mathematics more powerful than the system to be safeguarded; thus there is no safeguard at all.”²⁸ While Gödel’s incompleteness theorems show that the formalism cannot provide robust *philosophical* foundations for mathematics, they do not invalidate the use of formal axiomatic and reductive set-theoretic approaches by mathematicians. Indeed, from a mathematical perspective, formalism can be seen as a great triumph insofar as it brought increased rigor, clarity, and unity to the subject and led to a number of fundamental and fascinating results — including Gödel’s theorems themselves! The failure of the attempt to reduce mathematics to its formal aspect does not imply that this aspect is eliminable; one must remain cognizant of mathematics’ game-like and symbol-dependent character in theorizing about it.

Two more perspectives warrant attention to round out the philosophical spectrum, though they traditionally have had fewer adherents than the other positions discussed above. Both of these positions are classified as versions of constructivism, as they share the conviction that mathematics is a human-dependent construction.²⁹ The first of these positions, intuitionism, arose at roughly the same time as logicism and formalism, and completes the list of the traditional alternatives to platonism. The central commitment of intuitionism is

²⁷Mac Lane 379. Note that Gödel and others have proven the consistency of weaker formal systems, including the sentential and predicate calculi.

²⁸Ernest 19. While there is widespread agreement on this point, a few philosophers (notably, Michael Detlefsen) contend that Gödel’s incompleteness results do not constitute a definitive answer to Hilbert’s second question (Detlefsen 345–6). However, given that logicians and formalists ultimately wish to consider the *entirety* of mathematics as a single axiomatic system, Gödel’s result seems to entail that a consistency proof would have to be non-mathematical, an unpalatable consequence for most.

²⁹One may argue that “human-dependent” is too strong a condition, as it rules out the possibility of intelligent non-human entities doing mathematics.

that “mathematics is an essentially languageless activity of the mind having its origin in the perception of a move of time.”³⁰ That is, our capacity to distinguish successive moments in time combined with our powers of abstraction gives rise to the integers.³¹ Grounding arithmetic upon our time sense leads the intuitionists to some unusual conclusions. The fact that many statements about the future seem to be neither true nor false but indeterminate helps motivate the intuitionist rejection of the law of the excluded middle. This means that intuitionists are not allowed to use proofs by contradiction in their mathematics.³² Because there is always room for more moments in the future but those future moments do not yet exist, intuitionists accept the idea of a potential infinity but reject absolute infinities.³³ These rejections mean that an intuitionist mathematics will have a restricted scope in comparison to mathematics associated with other philosophical viewpoints. The primary complaint against intuitionism is that these restrictions entail the unacceptable loss of many important results of classical mathematics, some of which play a pivotal role in our most powerful scientific theories.³⁴ Another criticism is that the central commitment of intuitionism makes mathematics mind dependent and leads to a problematic relativism; some possible constructivist responses against charges of psychologism will be considered later in the chapter. While most see these objections as sufficient grounds for rejecting intuitionism, some devoted individuals continue to do mathematics according to intuitionist assumptions. Most mathematicians will agree that constructive proofs are powerful and can give insights which non-constructive proofs may fail to.³⁵ However, this merely provides a reason to favour constructive proofs

³⁰L.E.J. Brouwer, *Brouwer’s Cambridge Lectures on Intuitionism*, Ed. D. van Dalen (Cambridge: Cambridge UP, 1981), 4–5.

³¹In his *Critique of Pure Reason*, Immanuel Kant theorizes that mathematical knowledge emanates from the pure *intuitions* of space and time that provide *a priori* form to our cognitions. Thus, intuitionism borrows both its central commitment and its name from Kant’s philosophy of mathematics (Brown 119–20). While Kant’s theory of mathematics is historically significant and influenced Frege, Hilbert, and Brouwer, it has almost entirely fallen out of favour.

³²John D. Barrow, *Pi in the Sky: Counting, Thinking, and Being* (Oxford: Clarendon, 1992), 185–6.

³³Brown 128.

³⁴Brown 134.

³⁵A non-intuitionist may demonstrate the existence of some mathematical object by proving the assumption of its non-existence leads to contradiction; such non-constructive proofs provide little insight into how one might arrive at an instance of such an object. A constructive proof, on the other hand, demonstrates existence by providing a direct method for constructing an explicit instance of the object in question. The Intermediate Value Theorem (If f is continuous on the interval $[a, b]$ and there is a C with $f(a) < C < f(b)$ then there exists a c such that $a < c < b$ and $f(c) = C$) can only be proved non-constructively: there is no way to prove the theorem by showing a method for finding c directly (Brown 131). Brown notes that not all constructive

whenever they are possible, not to rely on them exclusively.

If intuitionism is rooted in psychology, then the second constructivist position considered here, social constructivism, is sociological in nature. Social constructivist theories hold that mathematics is a cultural institution, though their commitment to this varies in degree. The weakest versions of social constructivism involve only an acknowledgment that at least some mathematical concepts have historical and cultural aspects, a generally noncontroversial thesis. The growing literature on the history of mathematics provides significant evidence in favour of weak social constructivism.³⁶ Indeed, one need look no further than intuitionists and non-intuitionists to find two groups who practice mathematics in different ways! On the other end of the spectrum, the strongest social constructivisms hold that mathematics exists *solely* by social convention. This extreme end of the spectrum is populated primarily by straw men, as it seems to allow the possibility that arbitrary assumptions are mathematically legitimate as long as they are backed by the consensus of some community. Thus, both intuitionism and social constructivism need to answer charges of relativism. However, on a positive note, these constructivist philosophies make some important observations about mathematics-as-practice, usefully shifting the traditional focus.

All of the above viewpoints have been deemed unacceptably flawed. However, each of them captures something important about contemporary mathematical practice. Mathematicians talk and act as though mathematical objects have an objective platonistic existence; while it seems to be theoretically possible to alter our language to eliminate reference to mathematical entities, implementing such a change is thoroughly impractical and unnecessary. Much mathematical work is highly symbolic and formal, and is conceived of as being part of an axiomatic system. And yet, mathematical progress is clearly not a straightforward linear unpacking of axiomatic implications but rather involves hunches, hypotheses, and trial and error, and only becomes rigorous and formalized at a later stage. What seems to be

proofs provide direct methods; for example, one may constructively prove the compositeness of an integer using Fermat's theorem without giving a prime factorization or indicating how we might arrive at one (Brown 133).

³⁶Imre Lakatos played a key role in the popularization of social constructivism. His *Proofs and Refutations* (1976) takes the form of a dialogue that mirrors the historical developments surrounding the concept POLYHEDRON. This work opposes the tradition by arguing that definitions should be seen not as the starting point of mathematical practice but rather the end result of a process of conceptual development.

required is a philosophical position that acknowledges and incorporates the various aspects of mathematical practice rather than detrimentally focusing on one exclusively. While years of disagreement between the traditional viewpoints have exposed apparent incompatibilities that have been seen as an impediment to an amalgamated perspective, metaphor provides a promising mechanism for attaining an integrated theory.

If metaphor is to do the desired work, the following two preconditions must hold true. First, the understanding of metaphor invoked will necessarily be a non-traditional one, as traditional theories hold that metaphors are eliminable. Second, any theory of mathematics that includes metaphor as a constitutive element must acknowledge that mathematics must be at least partially human-dependent, to the extent that metaphors are human-dependent. As has been discussed above, these two ideas have been around for several hundred years but were not significantly developed until the twentieth century, and they remain controversial today. This helps explain why few authors have explored the possibility of metaphorical explanations of mathematics, and why such views did not emerge until recently. I consider two such theories below. Lakoff and Núñez's CMT-based theory of embodied mathematics is arguably the most ambitious and successful attempt to include metaphor as a constitutive element in a philosophy of mathematics. Their work is very promising, but it is still in its early stages and is not beyond criticism and improvement. Yablo's mathematical figuralism takes a different approach to incorporating metaphor, one that involves the idea of fictional objects. Yablo's theory initially provides contrast to Lakoff and Núñez's, but I ultimately argue that there is an underlying compatibility between Lakovian embodied mathematics and certain varieties of mathematical fictionalism.

4.2 Conceptual Metaphor Theory and Embodied Mathematics

Thanks to CMT's focus on the conceptual, discussion of mathematics has been a part of the project since its inception.³⁷ However, it is not until 20 years into the CMT project that

³⁷See, for example, *Metaphors We Live By*, pages 218–22, and *Women, Fire, and Dangerous Things*, chapter 20.

mathematics is given a thorough treatment in *Where Mathematics Comes From*. This book begins with the question “Exactly what mechanisms of the human brain and mind allow human beings to formulate mathematical ideas and reason mathematically?”³⁸ By taking this as the central question of their inquiry, Lakoff and Núñez invert the traditional approach, which demands solid objective mathematical foundations before complicating matters with considerations of fallible and idiosyncratic human experience and behaviour. Explaining the role of the brain in observed mathematical practices is an empirical undertaking, not a matter of *a priori* philosophical or mathematical theorizing. However, while *Where Mathematics Comes From* is primarily a work in theoretical cognitive science, the authors do also explicitly explore many immediate philosophical consequences of their research. In particular, they require their theory to account for the precision, consistency, stability, cross-cultural understandability, symbolizability, calculability, and descriptive and predictive effectiveness of mathematics.³⁹ My primary interest is not in whether the speculative details of their “mathematical idea analysis” are accurate but in the ramifications of their approach in general; thus, I will limit discussion of specific conceptual metaphors below, invoking them only as necessary to support the metaphorical approach generally.

Lakoff and Núñez’s account of mathematical concepts is founded squarely upon CMT: worldly perceptual stimuli interact with our genetically determined mental and bodily structures and capacities to yield direct experiences. Correlations between these direct experiences give rise to the most-basic conceptual metaphors which allow us to understand one experience in terms of another, thereby producing a new level of indirect experiences. These resultant experiences are then available as inputs for further metaphorical mappings and thus the cycle continues, increasing abstraction and complexity of both experience and metaphor as the layers stack. It is worth considering each of these stages in more detail. First, the biological capacities underpinning mathematical reasoning. There is a growing body of biological evidence regarding numerical competency. Studies targeting the capabilities of human infants and various non-human animal species are of particular interest as they provide evidence that some numerical abilities are ancient and genetically hardwired. Experiments indicate that

³⁸Lakoff and Núñez, *Where Mathematics Comes From* 1.

³⁹Lakoff and Núñez, *Where Mathematics Comes From* 50.

a wide range of animals have rudimentary arithmetical abilities. It is perhaps unsurprising that chimpanzees possess a fairly wide range of numerical capabilities, from discriminating the sizes of small collections of objects to arithmetic with small integers and simple fractions.⁴⁰ Somewhat more surprising is the fact that there is evidence that many mammals and birds — including rats, cats, raccoons, lions, dolphins, parrots, and pigeons — possess similar capabilities when it comes to small integers.⁴¹ It has been shown that angelfish are able to reliably discriminate quantities up to 4, showing that numerical capabilities are not confined to higher vertebrates.⁴² And, astoundingly, research involving bees and mealworms suggests that numerical discrimination abilities extend even to invertebrates.⁴³ The ability to numerically discriminate seems clearly advantageous from an evolutionary standpoint insofar as it helps individuals optimize outcomes when confronted with a choice between differently sized food sources, breeding groups, or defense organizations. This has led Dehaene to hypothesize the existence of a genetically determined “number sense” that explains the above animal results.⁴⁴

Experiments suggest that this number sense also exists in infant humans, lending credence to the idea that these capabilities are genetically determined.⁴⁵ Prior to 1980, the majority of psychologists subscribed to the Piagetian view that every newborn is a *tabula rasa*, mathematically speaking.⁴⁶ The earliest studies investigating the numerical capacities of babies contradicted this viewpoint by demonstrating that infants are able to discriminate between visual representations of 2 and 3 a few days after birth.⁴⁷ These experiments involved repeatedly showing infants slides with two objects of varying location, size, and identity on them until habituation was achieved, then presenting a slide with three objects. The atten-

⁴⁰Stanislas Dehaene, *The Number Sense: How the Mind Creates Mathematics*, 2nd ed. (New York: Oxford UP, 2011), 14.

⁴¹Lakoff and Núñez, *Where Mathematics Comes From* 21 and Dehaene, Chapter 1. Note that the ability to add fractions has only been observed in primates to date (Dehaene 14).

⁴²Luiz M. Gómez-Laplaza and Robert Gerlai, “Spontaneous discrimination of small quantities: shoaling preferences in angelfish (*Pterophyllum scalare*),” *Animal Cognition* 14.4 (2011): 565–74.

⁴³See Dacke and Srinivasan (2008) and Carazo et al. (2009).

⁴⁴Dehaene 28.

⁴⁵Lakoff and Núñez refer to these inborn abilities as “innate arithmetic” (*Where Mathematics Comes From* 19). I am reluctant to adopt this terminology, as the word “innate” has a variety of philosophical connotations that I expressly do not wish to invoke.

⁴⁶Dehaene 31.

⁴⁷Lakoff and Núñez, *Where Mathematics Comes From* 15.

tion fixation duration of the infants was significantly longer after the change, indicating a discrimination between the two stimuli that was attributed to a sensitivity to numerosity.⁴⁸ Subsequent experiments showed that newborns can likewise discriminate between auditory stimuli consisting of two and three syllables and, more significantly, that when six-month olds are simultaneously presented with one sequence of tones and two visual representations of different quantities, they consistently attend longer to the slide whose numerosity matches the auditory stimulus. This latter study provides strong evidence that our inborn sensitivity to numerosity is abstract and amodal rather than tied to visual or auditory perception, that is, “that the child really perceives numbers rather than auditory patterns or geometrical configurations of objects.”⁴⁹

The numerical competency of preverbal infants is not limited to mere quantity discrimination, but also seems to include some rudimentary arithmetic. In the early 1990s, it was shown that 4- and 5-month old babies possess some understanding of the addition and subtraction of small numbers. Researchers presented the infants with a scene where two distinct objects were placed behind a screen and then the screen was dropped, revealing either one, two, or three objects. It was observed that babies attended significantly longer to the two impossible scenarios than to the possible one, leading to the conclusion that the infants had expected to see two objects and were surprised or puzzled by the outcomes that conflicted with this expectation.⁵⁰ These studies and others show that infants possess arithmetic capabilities far beyond the expectations of previous generations of researchers. Though these results are significant and amazing, the extreme limitations of infant mathematics must be stressed. Infant arithmetic does not seem to extend beyond cardinality recognition, sums, and differences involving the integers between 1 and 4, and, significantly, does not even include an understanding of an ordering on those integers.⁵¹ These capabilities are more limited than those of adult chimpanzees, let alone adult humans.⁵² It seems our inborn powers are insufficient

⁴⁸Dehaene 38.

⁴⁹Dehaene 40. Though he rejects the view, Dehaene concedes that “[w]hile waiting for conclusive experiments with younger children, it remains possible to maintain that learning, rather than brain maturation, is responsible for the baby’s knowledge of numerical correspondence between sensory modalities” (50).

⁵⁰Dehaene 42.

⁵¹Dehaene 51.

⁵²Dehaene 45.

to handle arithmetic with large integers, let alone transfinite number theory, tensor calculus, and the rest of the gamut of human mathematics. Experiences and bodily developments must make a contribution.

It is clear that, as they age and develop, humans eventually acquire a wide range of mathematical knowledge and skills, most of which are connected to their capacity for symbol and language use, and the instruction they are thereby able to receive from their parents and teachers. A relevant question that arises is this: what is the connexion between inborn infant mathematics and symbolic adult mathematics? Some, such as extreme social constructivists, may hold that any correspondences between infant arithmetic and adult mathematics within contemporary individuals are coincidental, and claim that the two capacities develop entirely independently of each other. Most people, however, would likely be inclined to posit a parsimonious connexion between the two. Some evidence in favour of such a position comes in the form of the human subitizing capacity — our ability to quickly and accurately discern the cardinality of small collections of discrete objects (up to about four), a capability which seems to differ from counting (understood as sequential pairing).⁵³ A controversial but plausible hypothesis is that subitizing in adults and quantity discrimination in infants utilize the same neural circuitry.⁵⁴ As one might expect, Lakoff and Núñez believe that there is a strong connexion between infant and adult mathematics, endorse the above hypothesis and, further, claim that cognitive mappings — particularly, though not exclusively, conceptual metaphor — provide the mechanism by which our sparse inborn arithmetical capabilities, image schemata, and various other protoconceptual mechanisms bring about the full range of human mathematics.⁵⁵

Lakoff and Núñez introduce a classification scheme for mathematical conceptual metaphors in *Where Mathematics Comes From* that differs from the one I presented in chapter 3. Their primary distinction is between grounding and linking metaphors. *Grounding metaphors* map inferences about everyday experiences onto abstract concepts, connecting our inborn arith-

⁵³Lakoff and Núñez, *Where Mathematics Comes From* 19.

⁵⁴Dehaene 57.

⁵⁵Recall, from chapter 3, that an image schema is a cognitive mechanism that interactively imposes form on perceptions, thereby providing protoconceptual structure that facilitates basic reasoning and agency in the physical world (Lakoff and Núñez, *Where Mathematics Comes From* 30–1).

metic capacities to our basic agential interactions with the world, for example. It seems clear that there will be some overlap between the class of grounding metaphors and the class of primary metaphors discussed in chapter 3. *Linking metaphors* bridge two branches of mathematics, forming links between abstract concepts.⁵⁶ Linking metaphors that privilege their source domain as fundamental form the subclass of *foundational metaphors*; such mappings lurk at the core of attempts to formally reduce one branch of mathematics to another.⁵⁷ *Extraneous metaphors* are those which “have nothing whatever to do with either the grounding of mathematics or the structure of mathematics itself ... [they] can be eliminated without any substantive change in the conceptual structure of mathematics.”⁵⁸ While they may aid visualization or reference, extraneous metaphors do not make a constitutive contribution to mathematics.⁵⁹ This classification is important insofar as it is a key structural element in Lakoff and Núñez’s writing and is the locus of some important criticisms of their theory. It is important to note that while there is a strong emphasis on conceptual metaphor in *Where Mathematics Comes From*, several other cognitive capacities also play constitutive roles in their theory — including, but not limited to, conceptual metonymy, conceptual-blending, and symbolization.⁶⁰

Lakoff and Núñez posit that four fundamental grounding metaphors work in concert, using our direct worldly experiences to enrich our inborn, pre-conceptual quantity discrimination abilities into a NUMBER concept capable of supporting arithmetic in its full richness. Recall the hypothesis from chapter 3 that primary metaphors arise through neural confluences, “the simultaneous activation of two distinct areas of our brains, each concerned with distinct as-

⁵⁶Lakoff and Núñez, *Where Mathematics Comes From* 52–3.

⁵⁷Lakoff and Núñez, *Where Mathematics Comes From* 100.

⁵⁸Lakoff and Núñez, *Where Mathematics Comes From* 52.

⁵⁹Mathematical language is filled with eliminable catachreses: the legs of a right triangle, rings, sheaves, wreath products, stair functions, telescoping sums, and so forth. The object of study in my M.Sc. thesis is the pinwheel tiling, a mathematical object so named because its triangular tiles occur in an uncountable number of rotational configurations as do the blades of the rotating children’s toy. Such metaphors typically do not penetrate deeper than the names and therefore do not infect the rigorous mathematical reasoning with their imprecision: no one thinks of the vertices of a right triangle as hips or feet!

⁶⁰These other cognitive elements are often overlooked and deserve more attention than Lakoff and Núñez have given them. For example, the Fundamental Metonymy of Algebra allows roles played to stand for individuals and is hypothesized to be *the* mechanism which “allows us to go from concrete (case by case) arithmetic to general algebraic thinking” (Lakoff and Núñez, *Where Mathematics Comes From* 74). Clearly, such an important abstracting mechanism warrants more than a mere half a page of discussion! Rectifying such deficiencies falls outside the scope of this dissertation on *metaphor* and mathematics.

pects of our experience.”⁶¹ The first of the arithmetic grounding metaphors, ARITHMETIC IS OBJECT COLLECTION, arises from regular and frequent correlations in our experience between perception and manipulation of groups of physical objects and basic quantity discrimination, addition, and subtraction; the multimodality of our inborn number sense provides numerical experiences not associated with any object collections that allows the source and target domains to be differentiated.⁶² It is hypothesized that ARITHMETIC IS OBJECT COLLECTION is established at a very young age, long before any mathematical training occurs.⁶³ Human experiences of collections of objects possess extensive image-schematic structure that facilitate our agential interactions with those collections. For example, a constitutive part of our protoconceptual understanding of collections of physical objects is that the introduction of a new object into a collection A results in a different collection B, not a different *kind* of thing altogether (a non-collection).⁶⁴ Because conceptual metaphors are inference preserving, the grounding metaphor ARITHMETIC IS OBJECT COLLECTION is able to enrich our paltry inborn arithmetic by importing reasoning structures pertaining to physical collections. The preservation of collectionhood under object introduction maps to the preservation of numberhood under addition, for example, giving rise to part of the mechanism of mathematical closure.

The metaphorically induced mechanism of mathematical closure is necessary to extend our arithmetic beyond the first three integers and, additionally, provides one of the clearest illustrations of conceptual metaphors creating target domain entities. Infants’ distinct experiences of 1, 2, and 3 and the connections between them, such as $2 + 1 = 3$ and $2 - 1 \neq 2$, constitute a stable, structured protoconceptual understanding of numbers. The metaphor ARITHMETIC IS OBJECT COLLECTION induces further distinct numerical experiences that extend

⁶¹Lakoff and Núñez, *Where Mathematics Comes From* 42.

⁶²Whereas our “affection sense” is only triggered by a fairly small set of relatively specific circumstances (being cuddled, for example), our number sense is almost constantly working; this is because these two mechanisms work quite differently in service of our evolutionary fitness. Thus, AFFECTION and WARMTH are experienced independently more often than NUMBER and OBJECT COLLECTION, so the disanalogy is better established in the former case; that is, the source and target domains are recognized as connected, but distinct. I speculate that the conflationary coactivations involved in ARITHMETIC IS OBJECT COLLECTION are so persistent that differentiation between these domains is typically incomplete, leading to the impression that the relationship between them is something stronger than metaphor.

⁶³Lakoff and Núñez, *Where Mathematics Comes From* 54–5.

⁶⁴Lakoff and Núñez, *Where Mathematics Comes From* 81. Notice the use of the word “into” in the above sentence exposes part of the image schematic structure involved; see page 39 of *Where Mathematics Comes From* for more discussion of the INTO schema.

our understanding to include the other positive integers. The reasoning seems go something like this: a baby can recognize 1 and 2 as numbers and 3 — the result of $1 + 2$ — as a number, and also can recognize that introducing one more grape to an existing handful of three grapes changes the collection of grapes. Thus, because they distinguish 1 and 3 as numbers, the metaphor maps the introduction of a grape to the sum $3 + 1$, and the outcome to a new, different number: 4. Two important qualifying comments are necessary. First, despite the descriptions contained in the previous two sentences, one must remember that a large portion of this development is hypothesized to take place in the first few months of life, long before the acquisition of symbolic numerals.⁶⁵ Additionally, the development of this metaphor does not occur in a void, but rather seems to require the assistance of “appropriate language, emotional support, and behaviour.”⁶⁶ Second, it should be made clear that, even in newborns, the estimating faculty that allows us to distinguish between groups of objects is active when presented with collections of any cardinality, including those larger than three; however, we do not have enough inborn structure to *consistently, precisely, and systematically* distinguish between experiences of larger groups at birth.⁶⁷ The grounding metaphor bestows these properties upon the larger integers by transferring structure derived from our understanding of collections of physical objects that extends the consistent, discrete structure of the small integers to our larger quantitative approximations. ARITHMETIC IS OBJECT COLLECTION *creates* the number zero once the understanding of collections is expanded to include the idea of an empty collection, the result of taking one object away from a collection of one object, for example.⁶⁸ Lakoff and Núñez also claim that ARITHMETIC IS OBJECT COLLECTION is responsible for imposing an ordering on the integers that comes from an ability to distinguish which of two collections is bigger, a cognitive mechanism with clear survival benefits.⁶⁹ The other three arithmetic grounding metaphors — ARITHMETIC IS OBJECT CONSTRUCTION, ARITHMETIC IS

⁶⁵Lakoff and Núñez, *Where Mathematics Comes From* 55.

⁶⁶Rafael Núñez, “Numbers and arithmetic: neither hardwired nor out there,” *Biological Theory* 4.1 (2009): 78. Dehaene suggests that studies of the (until recently) isolated Mundurukú and Pirahã peoples show that gaps in this cultural scaffolding may result in an incomplete acquisition of ARITHMETIC IS OBJECT COLLECTION (263).

⁶⁷Lakoff and Núñez, *Where Mathematics Comes From* 51. Lakoff and Núñez refer to this ability to estimate quantities as the *numerosity capacity*.

⁶⁸Lakoff and Núñez, *Where Mathematics Comes From* 64.

⁶⁹Lakoff and Núñez, *Where Mathematics Comes From* 56.

USE OF A MEASURING STICK, and ARITHMETIC IS MOTION ALONG A PATH — systematically extend ARITHMETIC IS OBJECT COLLECTION in turn, allowing for the creation of fractional, irrational, and negative numbers respectively. These four are not the only grounding metaphors for the entirety of mathematics; one other example discussed in the book is CLASSES ARE CONTAINERS as a grounding metaphor in set theory.⁷⁰

Whereas grounding metaphors have extensively structured sensorimotor source domains, linking metaphors are not directly grounded in this way: they are maps between two abstract mathematical domains. Linking metaphors not only allow modeling to occur between previously established branches of mathematics, but can even create entire branches of mathematics, such as trigonometry.⁷¹ The majority of the conceptual metaphors presented in *Where Mathematics Comes From* are linking metaphors.⁷² One key example of a linking metaphor is NUMBERS ARE POINTS ON A LINE, which connects arithmetic and geometry. Descartes’ development of this metaphor blended the two domains together and laid the necessary foundations for the development of calculus.⁷³ Whereas grounding metaphors are typically acquired automatically early in life thanks to our genetic and social programming, linking metaphors arise through creative effort and require significant instruction if they are to be attained; because of this, there are many mathematical linking metaphors that only trained mathematicians possess.⁷⁴ Likewise, not every culture possesses the same linking metaphors; for example, our pre-Cartesian ancestors did not possess NUMBERS ARE POINTS ON A LINE, and, apparently, neither do the Yupno people of Papua New Guinea.⁷⁵ Linking metaphors are a crucial “part of

⁷⁰Lakoff and Núñez, *Where Mathematics Comes From* 123.

⁷¹Lakoff and Núñez, *Where Mathematics Comes From* 150. Discussion of the linking metaphors giving rise to trigonometry can be found in their Case Study 1, pages 383–98.

⁷²Lakoff and Núñez, *Where Mathematics Comes From* 53.

⁷³Recall from chapter 3 that a conceptual blend, as conceived by Fauconnier and Turner, involves the superimposition of conceptual structures connected by fixed correspondences, resulting in the creation of hybrid structures (Fauconnier and Turner 47). A *metaphorical blend* is a conceptual blend where a conceptual metaphor constitutes the fixed correspondences. Though Lakoff and Núñez allege that both metaphorical and non-metaphorical blends play an important role in mathematics, they provide no explicit non-metaphorical examples (Lakoff and Núñez, *Where Mathematics Comes From* 48). For an interesting discussion of the complementarity of conceptual blending theory and CMT, see Grady, Oakley, and Coulson (1999).

⁷⁴For example, a person requires at least a basic understanding of mathematical group theory in order to possess the ROTATION GROUP METAPHOR which maps group elements to geometric transformations (Lakoff and Núñez, *Where Mathematics Comes From* 116).

⁷⁵Rafael Núñez, Kensey Cooperrider, and Jürg Wassmann, “Number concepts without number lines in an indigenous group of Papua New Guinea,” *PLoS ONE* 7.4 (2012): 8. While neither of these example cultures have a full-fledged number line concept because they are lacking the linking metaphor, Núñez’s

the fabric of mathematics” insofar as they are responsible for making mathematics coherently unified across its several branches.⁷⁶

One other specific metaphor requires attention here. The collaboration between Lakoff and Núñez was initiated because Núñez’s preliminary research on INFINITY led him to hypothesize that metaphor may play a necessary role in conceptualizing it.⁷⁷ While *Where Mathematics Comes From* turned out to be far more than a discussion of INFINITY, it does remain a central topic of the book. Lakoff and Núñez claim that a single metaphor, the Basic Metaphor of Infinity (BMI), provides the basis for all mathematical conceptions of infinity. This metaphor is rooted in the aspectual system we use in conceptualizing events or processes. In particular, the BMI maps completed iterative actions (that is, actions with a perfective aspect) to indefinitely perpetual iterative actions (actions with an imperfective aspect). The main effect of this mapping is to transfer the idea of the termination of a process resulting in a unique final state from the perfective domain into the imperfective domain; in the latter, actions conceivably go on forever so there is no corresponding structure to map on to. Thus, the BMI creates actual infinity as the metaphorical end result of a potentially infinite process.⁷⁸ Neither the source nor the target domain of the BMI is inherently numerical; to obtain the various mathematical versions of infinity (for example, the final counting number or the edge of the Euclidean plane) the target domain of the BMI must be restricted to a specific mathematical imperfect iterative process, such as reciting the unending sequence of positive integers.⁷⁹ There are two particular reasons for mentioning the BMI here. First, arguably one of the most accessible and compelling arguments that metaphor plays an ineliminable role in mathematics is that it seems to be required for an understanding of infinity, among the most abstract and non-direct concepts in the human repertoire; it is thus perhaps no surprise that extensive discussions of the BMI feature prominently in *Where Mathematics Comes From*. Second, it is not clear what kind of conceptual metaphor the BMI is. It seems

findings suggest that the Yupnos may also be lacking portions of the ARITHMETIC IS MOTION ALONG A PATH and ARITHMETIC IS USE OF A MEASURING STICK grounding metaphors. However, as Núñez does not even use the word “metaphor” in the above article, it is difficult to interpret exactly how different he found the Yupnos’ suite of conceptual metaphors to be from our own.

⁷⁶Lakoff and Núñez, *Where Mathematics Comes From* 150.

⁷⁷Lakoff and Núñez, *Where Mathematics Comes From* xii.

⁷⁸Lakoff and Núñez, *Where Mathematics Comes From* 160.

⁷⁹Lakoff and Núñez, *Where Mathematics Comes From* 165.

like a linking metaphor insofar as it is the connecting thread running between the conceptions of infinity arising in different mathematical domains.⁸⁰ It also seems like a grounding metaphor in that its ultimate source is the sensorimotor aspectual system.⁸¹ Pursuing this point further, as noted above the general statement of the BMI is not inherently numerical and is invoked non-mathematically; thus, a third possibility is that it is neither a grounding nor a linking metaphor.⁸² Why this observation is noteworthy will become apparent in the criticisms below.

For Lakoff and Núñez, “[m]athematics is a product of the neural capacities of our brains, the nature of our bodies, our evolution, our environment, and our long social and cultural history.”⁸³ As such, embodied mathematics can be seen as a variety of constructivism, albeit one that is empirically based and descriptive rather than prescriptive. It is conceptual metaphor that sets this theory apart from its traditional predecessors, binding the above factors together and allowing mathematics to climb to ever loftier abstract heights while tethering it to our most basic human capacities and experiences. It is this metaphorical piggybacking of our rudimentary mathematical capacities upon our even more fundamental sensorimotor systems that establishes embodied mathematics’ mathematics-as-practice orientation. Dehaene’s experimental findings support the Lakovian view: “we use brain circuits to accomplish mathematical tasks that also serve to guide our hands and eyes in space — circuits that are present in the monkey brain, and certainly did not evolve for mathematics, but have been preempted and put to use in a different domain.”⁸⁴ Mathematics is something we have evolved to *do* to better function in the world.

The recognition that certain connexions in mathematics are metaphorical rather than literal allows embodied mathematics to incorporate traditional insights while circumventing many long-standing difficulties; I will return to discuss this aspect of the theory in the last section of the chapter. However, as we saw in chapter 3, avoiding traditional problems

⁸⁰Lakoff and Núñez, *Where Mathematics Comes From* 161.

⁸¹Lakoff and Núñez, *Where Mathematics Comes From* 156.

⁸²The BMI is clearly not an extraneous metaphor, as it cannot be eliminated without making a substantive change to the conceptual structure of mathematics (Lakoff and Núñez, *Where Mathematics Comes From* 53).

⁸³*Where Mathematics Comes From* 9.

⁸⁴Dehaene 246. Dehaene uses the term *neuronal recycling* rather than referring to the piggybacking phenomenon as conceptual metaphor.

does not place CMT beyond scrutiny: the theory does not seem as firmly established as Lakoff would have us believe. Likewise, embodied mathematics is a controversial theory, and has been criticized by mathematicians and philosophers alike. Most published reviews target *Where Mathematics Comes From* as a stand-alone work rather than as a fragment of the Lakovian oeuvre upon which it thoroughly depends. This may contribute to the fundamental misunderstandings behind many of the criticisms that cause them to fail. On the other hand, some criticisms of *Where Mathematics Comes From* are consistent with those directed at CMT generally despite their detachment from the greater corpus, suggesting that these objections may be more substantive. Considering a selection of criticisms will help expose the strengths and weaknesses of embodied mathematics, and suggest directions for future development.

Embodied mathematics is subject to most of the frequently accurate yet nonfatal shallow criticisms directed at CMT discussed in chapter 3 above. Lakoff and Núñez are not always the best advocates for their own theory. Their writing can be nicely accessible, but is often stylistically off-putting in various ways. At times, Lakoff and Núñez seem to hubristically overstate the novelty of their mathematical interpretations.⁸⁵ They have a tendency to caricature rival positions, challenging straw men rather than the nuanced views of specific authors; as Presmeg puts it:

I had the feeling that at times, in their zeal to destroy the myth [of the Romance of Mathematics], they were tilting at windmills... For instance, in the section titled ‘A Question of Faith: Does Mathematics Exist Outside Us?’ the authors appeared to be setting up arguments solely for the purpose of tearing them down (pp. 342-343). The resulting picture appeared to be a caricature, and I questioned whether anyone in fact believed the Romance view in the extreme form in which it was portrayed.⁸⁶

The first printing of the book contains a number of acknowledged mathematical errors, though these were corrected with published errata and in subsequent printings.⁸⁷ In general, *Where Mathematics Comes From* comes across as slipperier than most works in the philosophy of

⁸⁵Gerald A. Goldin, “Review: Counting on the Metaphorical,” *Nature* 413.6851 (2001): 19.

⁸⁶Norma Presmeg, “Review — Mathematical Idea Analysis: A Science of Embodied Mathematics,” *Journal for Research in Mathematics Education* 33.1 (2002): 61.

⁸⁷George Lakoff and Rafael Núñez, “Reply to Bonnie Gold’s Review of *Where Mathematics Comes From*,” *MathDL*, The Mathematical Association of America, 5 May 2009.

mathematics; while a certain degree of looseness is to be expected and tolerated given the authors' beliefs about metaphor and thought, even Lakoff and Núñez concede that "there are indeed some passages in which we stated things in a sloppy way and we apologize."⁸⁸ There are many passages in the book where details are scant and elaboration or, at the very least, specific references — particularly to other works in the CMT corpus and evidential sources — would be most welcome. While none of these observations constitute a devastating objection to embodied mathematics, they can affect readers with a dismissive predisposition, preventing them from seeing past the presentational problems to give the underlying theory a fair assessment.

Even if one is able to successfully exercise philosophical charity and avoid becoming biased against Lakoff and Núñez on account of their atypical style, the scantness of certain key passages in the book certainly contributes to bringing about the misunderstandings that plague many of the objections raised against embodied mathematics. In an uncharacteristic move, Lakoff responds to a review of *Where Mathematics Comes From* by the mathematician Bonnie Gold, observing that several of her comments indicate a fundamental misunderstanding of their position.⁸⁹ In particular, they take Gold's claim that their arguments in favor of human mathematics "have little direct connection with the rest of the book" and the total absence of the word "embodiment" from her review as evidence that she has fundamentally missed the central importance of embodiment in their theory.⁹⁰ By stripping the embodied human element from their theory, Gold ends up misinterpreting their cognitive analyses as mathematical ones and viewing conceptual metaphors as "essentially isomorphisms," and, accordingly, puts forth a variety of fundamentally flawed criticisms of the specifics of their idea analysis.⁹¹ Though Lakoff and Núñez's only published response targets Gold's review specifically, it is clear that other critics have also fundamentally misunderstood their position.

⁸⁸"Reply to Bonnie Gold's Review."

⁸⁹Lakoff and Núñez, "Reply to Bonnie Gold's Review."

⁹⁰Lakoff and Núñez, "Reply to Bonnie Gold's Review." Presmeg's speculation that Lakoff and Núñez's discussion of neuroscience and infant arithmetic is ultimately distracting and unnecessary seems to betray a similar misunderstanding on her part (Presmeg 60).

⁹¹Bonnie Gold, "Read This! The MAA Online book review column: Where Mathematics Comes From: How the Embodied Mind Brings Mathematics Into Being," *MathDL*, The Mathematical Association of America, 5 May 2009. It should be noted that the conceptual metaphor CONCEPTUAL METAPHORS ARE ISOMORPHISMS may be a fruitful one in a different context, when one is trying to understand conceptual metaphor in terms of mathematics rather than vice versa; for more on such ideas, see chapter 5.

Whereas most criticisms of Lakoff and Núñez come from book reviews written by mathematicians, the philosophers Parsons and Brown are among the few to have published a critical article responding to *Where Mathematics Comes From*. Their article contains a variety of criticisms motivated by Lakoff and Núñez’s rejection of mathematical platonism and the “Romance of Mathematics.” Parsons and Brown interpret the anti-platonic argument in the following way:

1. Human mathematical thought is based on conceptual metaphor (i.e., its statements attribute physical properties to numbers and sets) (empirical result).
2. Human mathematical thought is true metaphorically but not literally (from 1).
3. If there are Platonic entities (numbers and sets), then statements describing them are literally true (assumption).
4. Therefore, it is not the case that human mathematical thoughts truly describe Platonic entities, even if they do exist (from 2,3).⁹²

Their objection focuses on the middle two statements of the argument:

This argument is fallacious because it depends on the claim that metaphorical statements cannot be used to express literal truths, which is patently false... Often the only way we can express a (real, literal) truth is by using a metaphor, by saying something that, taken literally, does not mean quite what we want to get across, but that we know will have the correct metaphorical meaning for our listeners.⁹³

While Parsons and Brown’s counterargument seems plausible at first glance, it is based upon an equivocation. Within the first premise, they transition from an understanding of metaphor as conceptual to a traditional linguistic understanding of metaphor. It is the parenthetical portion of the first premise that introduces the error: while the *statements* of mathematics do metaphorically attribute physical properties to numbers and sets, this is merely a symptom of conceptual metaphor, not the whole of it. This misunderstanding demonstrates how easily one’s own paradigm can creep in and subvert a genuine attempt to understand embodied mathematics in its own paradigmatic milieu: Parsons and Brown clearly and explicitly note

⁹²Glenn G. Parsons and James Robert Brown, “Platonism, Metaphor, and Mathematics,” *Dialogue* 43.1 (2004): 52.

⁹³Parsons and Brown 52.

in the first section of their paper that “[c]onceptual metaphors...are not to be understood as linguistic phenomena.”⁹⁴

Given the equivocation in premise 1, it becomes more difficult to interpret the already obscure notion of metaphorical truth introduced in premise 2. Throughout their paper, Parsons and Brown rely on a traditional interpretation which can be approximated thusly: a metaphorical truth is an instance of non-literal language used to express a different literal truth. In this interpretation, metaphorical truth is secondary, as it is parasitically dependent on literal truth. It is not far from such a traditional interpretation to the once-popular idea that metaphor constitutes an occasionally useful but primarily obfuscating deviant use of language that can be — and usually *ought* to be — carefully paraphrased as the equivalent literal statement.⁹⁵ The traditional interpretation is incompatible with the conceptual metaphor interpretation: a metaphorical truth is a truth expressed in language reflecting concepts structured by non-extraneous conceptual metaphors.⁹⁶ For Lakoff and Núñez, our concepts are not mere imperfect mirrors of objective reality. They do depend on our experiences of the world and its regularities, but are ultimately active human constructions as opposed to passive resemblances. All truths depend on our embodied concepts, and most — if not all — concepts are partially structured by conceptual metaphor. Therefore, most truths are at least partially metaphorical under the conceptual interpretation. This is clearly not the case in the traditional interpretation, where the majority of truths must be literal. Lakoff and Núñez’s point is that there is an irreconcilable disparity between the platonist understanding of concept formation (where concepts approximate objective, human-independent structures) and the embodied understanding of concept formation (where all concepts are thoroughly human, arising from necessarily embodied experiences).⁹⁷ Parsons and Brown’s equivocation reveals a misunderstanding of the theory of conceptual metaphor that masks this disparity,

⁹⁴Parsons and Brown 49.

⁹⁵Lakoff and Johnson, *Philosophy in the Flesh* 119

⁹⁶As far as I am aware, Lakoff and Núñez never use the term “metaphorical truth” in their work, but I believe the above interpretation is satisfactorily consistent with their view.

⁹⁷Lakoff and Núñez do concede that the idea that platonic entities exist is not itself inconsistent with embodied mathematics, though they argue that there are ontological reasons to believe their existence untenable; regardless, given the impossibility of accessing such entities, positing their existence is entirely superfluous (*Where Mathematics Comes From* 342).

causing their central counterargument to fail.⁹⁸

As was noted in chapter 3, Lakoff and Johnson have consistently maintained that a position known at first as experientialism and later as embodied realism is an integral component of their theory, claiming that “you cannot simply peel off a theory of conceptual metaphor from its grounding in embodied meaning and thought. You cannot give an adequate account of conceptual metaphor and other imaginative structures of understanding without recognizing some form of embodied realism.”⁹⁹ A brief summary of the embodied realist position, distilled from the entire Lakovian corpus, will augment the paltry account provided in *Where Mathematics Comes From* and help overcome misunderstandings like those experienced by the critics cited above. For the embodied realist, the basis of all human cognition is *embodied experience*. Whether you are seeing a tree *with your eyes*, hearing a friend call your name *with your ears*, feeling a pain *in your back*, or imagining a unicorn *in your head*, your human body makes an ineliminable contribution. Experiences are interactive gestalts: they arise as unified wholes that are only later teased apart through interpretation. Importantly, embodied experiences are conceived as incorporating contributions from both the world and the capacities and faculties of our bodies, though these aspects cannot be clearly and fully disentangled from each other. This entanglement means there is no “ontological chasm” between subject and object (or mind and body, if you prefer) to be explained away.¹⁰⁰ Embodied realism is committed to the existence of a stable, mind-independent world that exhibits regularities, an ultimate material basis for reality.¹⁰¹ However, there is no way for us to transcend our embodiment and gain unmediated access to that structure: there is no such thing as a disembodied experience. Conversely, every body is situated in some worldly environment or other at all times. As Mark Johnson puts it, “our structured experience is an organism-environment interaction in which both poles are altered and transformed through an ongoing historical process.”¹⁰²

One important consequence of embodied realism is that it “requires us to give up the

⁹⁸ An extended version of this response to Parsons and Brown can be found in Postnikoff (2008).

⁹⁹ Johnson and Lakoff, “Why cognitive linguistics requires embodied realism” 245.

¹⁰⁰ Lakoff and Johnson, *Philosophy in the Flesh* 93–4.

¹⁰¹ Lakoff and Johnson, *Philosophy in the Flesh* 90, 110.

¹⁰² *The Body in the Mind* 207.

illusion that there exists a unique correct description of any situation.”¹⁰³ A key example will help clarify this claim. Lakoff and Johnson observe that humans exhibit two different levels of understanding when it comes to colours: a phenomenological level and a neurobiological level. At the phenomenological level, we experience colours as inherent properties of objects, leading us to make assertions like “That emerald is green.” At the neurobiological level, we understand colours as neural activations resulting from interactions between cells in our retinas and reflected light of specific wavelengths. Thus, “[a]t the neural level, green is a multiplace interactional property, while at the phenomenological level, green is a one-place predicate characterizing a property that inheres in an object.”¹⁰⁴ We take statements originating from both of these levels to be valuable and true, and yet they often contradict each other. Privileging one of these levels of understanding and subjugating the other does not adequately resolve the conflict between them, as such a move either hampers our ability to communicate or does injury to our notion of TRUTH, or both. Embodied realism bypasses this dilemma by positing that the truth of a sentence is relative to understanding (rather than some objective, human-independent state of the world) and thus acknowledges the legitimacy of both levels of understanding without contradiction, a move not possible within most traditional theories.¹⁰⁵

It is the embodied realist dimension of the Lakovian theory that defends it against charges of psychologism and relativism. Such objections are frequently raised against constructivist and empiricist theories of mathematics, and some may worry that viewpoints that prioritize mathematics-as-practice are vulnerable to these criticisms as well. In particular, the Lakovian posit that truth is *relative* to understanding seems to make embodied mathematics a target for antirelativistic objections. The most noteworthy champion of antipsychologism is Frege. One of the main theses of his *Grundlagen der Arithmetik* is “that mathematics and logic are not part of psychology, and that the objects and laws of mathematics and logic are not defined, illuminated, proven true, or explained by psychological observations and results.”¹⁰⁶ Frege contends that taking mathematics to be essentially psychological leads to

¹⁰³Lakoff and Johnson, *Philosophy in the Flesh* 109.

¹⁰⁴Lakoff and Johnson, *Philosophy in the Flesh* 105.

¹⁰⁵Lakoff and Johnson, *Philosophy in the Flesh* 106.

¹⁰⁶Martin Kusch, “Psychologism,” *Stanford Encyclopedia of Philosophy*, 14 Apr. 2012.

a variety of problematic results, chief among which is that it makes mathematical truth subjective and mind-dependent. This is antithetical to the certainty and universality that seem to underlie our deepest mathematical understandings. While contemporary practice-oriented views do tend to discuss the psychological states people possess when doing mathematics, this does not necessarily entail they hold mathematics to be *constitutively* psychological; that is, regardless of one's philosophical disposition it is presumably now non-controversial that people use their brains when doing mathematics. Even Frege acknowledges that mathematics involves thinking and that studying such practices is permissible: "It may, of course, serve some purpose to investigate the ideas and changes of ideas which occur during the course of mathematical thinking."¹⁰⁷ That being said, many practice-oriented positions are unrepentantly psychologistic — including Lakoff and Núñez's — and owe some sort of response to Frege.

The following crude rendering of Frege's antipsychologistic argument provides context for Lakoff's response:

If mathematics is constitutively psychological, it is subjective. If mathematics is subjective, it is not objective. If it is not objective, mathematics is radically relative. But this is an absurdity and an affront. Therefore, mathematics cannot be psychological.

Lakoff claims that Frege's argument contains a fatal error. It falsely concludes that because the subjective belongs to the realm of the psychological, the realm of the psychological is entirely subjective.¹⁰⁸ This erroneous argument reflects the primitive understanding of psychology that was available to Frege; it is almost certainly not simply an instance of one of the foremost logicians of all time bungling categorical containment like a novice. To help put Frege's understanding of psychology in context, consider that his *Grundlagen* (1884) was published over a decade before Freud's first musings on the unconscious in 1895.¹⁰⁹ When coupled with an appropriate contemporary understanding of psychology, Lakoff's observation provides the most promising general line of defense against Fregean antipsychologism. Frege's

¹⁰⁷Gottlob Frege, *The Foundations of Mathematics*, Trans. J.L. Austin, 2nd ed. (New York: Harper and Brothers, 1953), xviii.4

¹⁰⁸Lakoff and Johnson, *Philosophy in the Flesh* 462.

¹⁰⁹David L. Smith, *Freud's Philosophy of the Unconscious* (Dordrecht, Kluwer: 1999), 54.

argument depends on the false dichotomy between objectivism and radical relativism that is antithetical to embodied realism; as Mark Johnson puts it:

we ought to reject the false dichotomy according to which there are two opposite and incompatible options: (a) Either there must be absolute, fixed value-neutral standards of rationality and knowledge, or else (b) we collapse into an “anything goes” relativism, in which there are no standards whatever, and there is no possibility for criticism.¹¹⁰

Embodied mathematics espouses not a radical “anything goes” relativism, but a tempered relativism constrained by ubiquitous commonalities in human biology and the Terran environment that provide a non-absolute yet solid basis for mathematics. Mathematics is neither ad hoc and groundless, nor grounded in human-independent absolutes, but is stably grounded internally by congruences and consistencies in human experience.

The detailed story that Lakoff and Núñez tell about the biological origins of mathematics that fends off charges of relativism has led some critics to voice a very different concern. Rather than objecting that embodied mathematics is problematically arbitrary, several authors worry that the Lakovian story is too specific. As Goldin puts it, “[Lakoff and Núñez] often seem to assume, quite unjustifiably, that each mathematical construct can be understood in only one such way — the one they have discovered — and that they have found the *real* metaphor from which the mathematics originates.”¹¹¹ If Goldin is correct, then embodied mathematics would appear to be inconsistent with CMT, which allows for multiple legitimate ways of understanding; moreover, this concern stands independent of whether one takes the individual metaphors in the mathematical idea analysis to be convincing or spurious.¹¹² However, there are good reasons to think that most objections of this type miss the mark. One of the main goals of *Where Mathematics Comes From* is to account for the perceived characteristic features of mathematics — universality, precision, stability, and so

¹¹⁰*The Body in the Mind* 196.

¹¹¹Goldin 19.

¹¹²Of course, many critics further argue that certain specific parts of the idea analysis are overly simplistic, or outright erroneous; see, for example, Parsons and Brown’s criticism of the Lakovian analysis of set theory (54–5). While I also find some parts of Lakoff and Núñez’s analysis unconvincing, I will not consider such objections here as my primary concern is with defending the CMT approach in general rather than their specific analyses. While Lakoff’s stylistic tone may suggest certainty and finality, embodied mathematics must be open to ongoing revision and amendment if it is to be an empirically responsible theory; Lakoff and Núñez even admit that their cognitive approach to mathematics is in its preliminary stages, suggesting that they would admit there is room for improvement and development (*Where Mathematics Comes From* 11).

forth — within the framework of CMT and embodied realism and therefore without recourse to human-independent mathematical entities. As such, the book focuses on describing the ubiquitous conceptual metaphors that provide the skeleton of understanding that grounds our collective mathematics rather than the cultural and individual idiosyncrasies that flavour that core understanding. Thus, while Lakoff and Núñez do contend that there exists a collection of conceptual metaphors that feature in the understanding of every mathematics user, they do not claim that our understanding is restricted to or exhausted by these metaphors. In their concluding summary, they explicitly note:

Human conceptual systems are not monolithic. They allow alternative versions of concepts and multiple metaphorical perspectives of many (though by no means all!) important aspects of our lives. Mathematics is every bit as conceptually rich as any other part of the human conceptual system. Moreover, mathematics allows for alternative visions and versions of concepts. There is not one notion of infinity but many, not one formal logic but tens of thousands, not one concept of number but a rich variety of alternatives, not one set theory or geometry or statistics but a wide range of them — all mathematics!¹¹³

It would seem that Lakoff and Núñez are only guilty of failing to adequately discuss individual and cultural variations in mathematical metaphors, not of ruling them out completely.

While this failure thus does not constitute a fatal inconsistency, it remains a deficiency in the presentation of the Lakovian theory of embodied mathematics that requires attention. Even conceding that the main focus of *Where Mathematics Comes From* is (and should be) to provide an explanation of the ubiquitous core of mathematical understanding that unfolds within all of us, it is somewhat staggering that so very little of the book is allotted to reminding the reader that variations can and do occur between individuals' understandings of mathematics, let alone to discussing the details of some of those variations. This deficiency seems to be at the root of a couple of telling criticisms which may provide guidance for clarification and emendation. For one, readers not immersed in CMT who are engaging with the text as a stand-alone work may perceive embodied mathematics as having a problematically naturalistic bias; Madden is one such critic:

In my opinion . . . a naturalistic approach should certainly not dismiss the way mathematicians share definitions with one another, understand and criticize one

¹¹³Lakoff and Núñez, *Where Mathematics Comes From* 379.

another's reasoning, and use a precise, if artificial logical language to put their ideas in writing so that those ideas can be judged by the world. Surely we can acknowledge a role for intuition without ignoring the ways that logic and conventional rigor support the kind of knowledge that mathematicians build and share.¹¹⁴

First, it should be said that Lakoff and Núñez do acknowledge the power and rigor of the formal approach, and find a place for it within the theory of embodied mathematics by way of the Formal Reduction Metaphor — more on this below. And yet, there is certainly much more to be said about the connexions between the more informal symbol, language, and diagram use that seems to constitute the majority of our mathematical practices and the mostly unconscious naturalistic ground of our mathematical ideas.¹¹⁵ Seen in this way, Madden's comment is a version of the most significant unresolved general criticism of CMT found in chapter 3: that Lakoff tends to inadequately discuss the relationship between the conceptual and the linguistic, subjugating the latter as merely symptomatic of the former. As such, the suggestions made in that chapter — that both semiotics and the work of Cornelia Müller provide promising approaches to rectifying the Lakovian conceptual/naturalistic bias — also hold here. One specific concern that needs attention is the thought that some conceptual metaphors, mathematical or otherwise, might be better understood as arising through conscious and intentional action rather than unconscious naturalistic processes. In particular, discussion of how CMT accommodates the speech act that Searle calls declaration seems like it could be quite fruitful. Declarations allow language users to invoke their authorial power to create genuinely novel linguistic or institutional facts; the mathematically necessary activities of naming, defining, and introducing new rules are all instances of declaration.¹¹⁶ Thus, acts of declaration provide one likely non-naturalistic mechanism for conceptual metaphor development.

A related criticism, raised by Presmeg, involves the way metaphors are classified in *Where Mathematics Comes From*:

¹¹⁴Madden 1186.

¹¹⁵The symbol, language, and diagram use in most mathematics is informal only in comparison to *Principia Mathematica* and other strict works of logic; even the most informal mathematics will usually still seem achingly formal to the layman.

¹¹⁶John Searle, "A taxonomy of illocutionary actions," *Expression and Meaning* (Cambridge: Cambridge UP, 1979), 16–20.

Lakoff and Nunez have undervalued what they call *extraneous* metaphors (p. 53). There are metaphors, sometimes idiosyncratic, that individual mathematicians or students may construct in their learning experiences, which are a powerful part of sense-making in those experiences... individually constructed metaphors are an issue that belongs with consideration of effective teaching and learning of mathematics, and in this field they cannot be relegated to the role of an epiphenomenon.¹¹⁷

Lakoff and Núñez do not make it clear whether the tripartite distinction between grounding, linking, and extraneous metaphors is to be understood as exhaustive or not; a careful reading of their descriptions of these classes suggests some mathematical metaphors may exist that do not fit well into any of the three (recall the observation that the BMI, arguably the most important metaphor in the book, presents classificational problems).¹¹⁸ Such metaphors would possess a non-mathematical yet non-basic source domain and make at least a minimal contribution to one's understanding of mathematics; as this seems to be precisely the kind of metaphor that Presmeg has in mind, I will refer to them as Presmegian metaphors for convenience. If a distinct fourth class of mathematical metaphors does exist then Lakoff and Núñez should have given it at least a brief mention. On the other hand, if the classification scheme is taken to be exhaustive, then any Presmegian metaphors would have to be classified as either grounding metaphors or extraneous metaphors.¹¹⁹ It is plausible that if an idiosyncratic metaphor with a non-mathematical source domain made a substantial contribution to a person's understanding that Lakoff and Núñez would classify it as a grounding metaphor even if the source domain was abstract and the grounding therefore indirect; their explanation of grounding metaphors is vague and few examples other than the four fundamental grounding metaphors of arithmetic are explicitly indicated in the text. However, if Presmeg is correct that Lakoff and Núñez would classify such metaphors as extraneous, then we seem forced to acknowledge that there are grades of extraneity, as Presmegian metaphors involve understanding in a way that facile catachrestic conveniences like "step function" do not.

¹¹⁷Presmeg 62.

¹¹⁸It would be nice to present a good example of such a metaphor, but, as mentioned previously, the nature of conceptual metaphor makes it difficult to arrive at such an example. While I might dream up any number of possible mappings, the process of "dreaming up" would tend to produce extraneous metaphors, which would fail to be examples.

¹¹⁹They would clearly not be linking metaphors: not only are the necessary conditions on this class clearly laid out (i.e., source and target domains both mathematical), but the majority of *Where Mathematics Comes From* is devoted to examples of linking metaphors that confirm the definition (53).

Moreover, combining this insight with the idea of non-naturalistic conceptual metaphor development mechanisms raised in the previous paragraph suggests that it may be occasionally possible for an initially shallow catachrestic metaphor to develop into something substantial, robust, and non-extraneous. This possibility suggests that a more-dynamic understanding of metaphor classification is warranted.¹²⁰

The idea that Lakvoian taxonomies miss out important dimensions of metaphor occurs elsewhere in the critical literature. Schiralli and Sinclair recognize two distinctions missed by Lakoff and Núñez that could significantly clarify the theory of embodied mathematics. First, Schiralli and Sinclair observe that “[*Where Mathematics Comes From*] does not differentiate the term ‘mathematics,’ nor indicate whether metaphor might function differently depending on whether one is *learning*, *doing*, or *using* mathematics.”¹²¹ That there may be significant differences between these modes seems plausible considering that concepts seem to be more rigid when being used and more plastic when being learned. In particular, Lakoff and Núñez do not discuss metaphors with mathematical source domains and non-mathematical target domains which may play an important role in mathematical modeling.¹²² If this distinction is found to be significant, Müller’s work on creating a robust diachronic understanding of metaphor to replace the living/dead distinction could provide useful insights. Second, Schiralli and Sinclair distinguish between conceptual mathematics (mathematics as a group practice, or discipline) and ideational mathematics (how individuals represent the public mathematical concepts to themselves).¹²³ The idea that there may be idiosyncratic variation in how individuals represent mathematical concepts is closely related to Presmeg’s criticism; indeed, Schiralli and Sinclair claim that

Lakoff and Núñez provided metaphorical pathways to the *concepts* of [conceptual mathematics], through grounding and linking metaphors, but these are not necessarily the same pathways that an individual will follow in creating his or her *conceptions* in [ideational mathematics]. Lakoff and Núñez do not distinguish [ideational mathematics] from [conceptual mathematics]. In fact, they imply that

¹²⁰It seems plausible that Müller could make a significant contribution here.

¹²¹Martin Schiralli and Nathalie Sinclair, “A Constructive Response to ‘Where Mathematics Comes From,’” *Educational Studies in Mathematics* 52.1 (2003): 81.

¹²²For further discussion and examples of such metaphors, see chapter 5.

¹²³Schiralli and Sinclair 81. Note that this strongly mirrors Müller’s distinction between the levels of system and use (12–3).

ideational and conceptual mathematics will be isomorphic.¹²⁴

To sum up, Schiralli and Sinclair “do not deny the power and pervasiveness of metaphor” but do worry that the picture of metaphor presented in *Where Mathematics Comes From* is problematically lacking in complexity, and emphasize that further empirical research is necessary to resolve these speculations.¹²⁵

Where Mathematics Comes From is an ambitious first attempt to explain mathematics within the Lakovian framework of CMT and embodied realism. While it is clear that there is much more work to be done, and that some of Lakoff and Núñez’s specific analyses are spurious, the core insight that conceptual metaphor plays a constitutive role in mathematical reasoning and provides a mechanism by which mathematics can be grounded in basic experiences and primitive biological capacities is both appealing and promising. A variety of criticisms have been proffered against the theory. Many objections fail because they fundamentally misunderstand the Lakovian position and revert to a traditional linguistic conception of metaphor. The telling criticisms are not devastating to embodied mathematics but do indicate important directions for necessary development. A great deal of concern could be alleviated simply through improved scholarship, particularly making explicit, clear, and frequent citations.¹²⁶ Clearly situating embodied mathematics with respect to the empirical studies underlying it, the earlier portions of the Lakovian corpus it depends upon, and other leading theories of mathematical development and practice would fortify the theory significantly. While addressing the first two of these strengthens the foundations of the theory and helps avoid misunderstanding, exploring the relationships between embodied mathematics and other theories may help fill in some of the lacunae. One of the key aims of this chapter is to perform some of this clarificatory work. Even if all the existing empirical evidence behind the theory were exposed, it seems a considerable amount of additional scientific research would be necessary to thoroughly substantiate the theorizing which has taken place; this dissertation indicates a few noteworthy recent results in the field, but otherwise does not aim

¹²⁴Schiralli and Sinclair 84.

¹²⁵Schiralli and Sinclair 88–9.

¹²⁶While some may rightly argue that *Where Mathematics Comes From* and many other books in the Lakovian oeuvre are written more for the layperson than the specialist, this neither invalidates nor appeases the specialists’ desire for a scholastically rigorous version of the theory.

to make a contribution in this regard. Whether one is persuaded that embodied mathematics shows promise or not, it is worth briefly considering some competing theories of metaphorical mathematics.

4.3 Yablo’s Mathematical Figuralism

While Lakoff and Núñez have produced the most systematic and comprehensive theory of mathematics-as-metaphorical thus far, they are not the only scholars to have explored such ideas. Some of the critics mentioned above — notably Schiralli and Sinclair — endorse the idea that metaphor plays a constitutive role in mathematics, even if they disagree with the particulars of Lakoff and Núñez’s approach. It is noteworthy that researchers in mathematics, in philosophy, and in mathematics education have independently published work supporting the idea that metaphor plays a more important role in mathematics than traditionally thought. Perhaps the most well known of these is George Pólya’s two-volume *Mathematics and Plausible Reasoning*. In this work, Pólya distinguishes demonstrative from plausible reasoning, claiming that though both play crucial and complementary roles in mathematics, the importance of the latter has frequently been eclipsed by the former; that is, students are taught to produce rigorous proofs and calculations more than creative conjectures. *Mathematics and Plausible Reasoning* aims to rectify the imbalance between the two types of mathematical reasoning. Though Pólya never uses the word “metaphor” in that work, analogical reasoning is considered as a key species of plausible reasoning.¹²⁷ Another mathematician who has considered the connexions between mathematics and metaphor is Yuri Manin, whose “Mathematics as Metaphor” suggests that mathematics be considered as a “specialized dialect of the natural language,” which is, as he understands it, fundamentally metaphorical.¹²⁸ Mathematics-education professor David Pimm has been arguing that “[metaphor and analogy] are as central to the expression of *mathematical* meaning as they are to the expression of meaning in natural language” since the late 1970s.¹²⁹ And philoso-

¹²⁷George Pólya, *Induction and Analogy in Mathematics* (Princeton: Princeton UP, 1954), 13.

¹²⁸Yuri Manin, “Mathematics as Metaphor,” *Proceedings of the International Congress of Mathematicians, August 21–29, 1990* (Tokyo: Springer-Verlag, 1991), 1666.

¹²⁹David Pimm, “Metaphor and Analogy in Mathematics,” *For the Learning of Mathematics* 1.3 (1981): 47. Pimm initially adopts a comparison view of metaphor, but shifts to a Lakovian approach in later work

pher and award-winning poet Jan Zwicky has written several works on the importance of metaphorical thought, including “Mathematical Analogy and Metaphorical Insight” in which she argues that mathematical and metaphorical insight are analogous in hopes of establishing the legitimacy of the latter to those who still believe metaphor to be an eliminable linguistic garnish.¹³⁰ However, after Lakoff and Núñez, it is arguably Stephen Yablo who makes the strongest philosophical case for metaphor being an integral part of mathematical practice.

In “Go Figure: A Path through Fictionalism,” Yablo sketches a theory of mathematics which he refers to as figuralism.¹³¹ His position takes its basic inspiration from the fictionalist insights of Hartry Field and others, but improves upon them by carefully integrating Kendall Walton’s account of representation as make-believe, among other elements. “Go Figure” begins with a puzzle: people routinely utter sentences whose truth seems to depend on the existence of entities they do not believe in. At least some mathematical sentences are prime examples of this predicament. Quine provides a list of three ways one might deal with such a sentence: paraphrase the sentence so that the result is free of the problematic commitment, stop uttering that sentence, or acknowledge the commitment to the questionable entities.¹³² Yablo claims that a fourth possibility lurks in Quine’s writing, though he did not explicitly include it on the list: understand the sentence as advanced in a make-believe or fictional spirit.¹³³ This response to the puzzle — fictionalism — has the advantage of allowing us to have our cake and eat it too: it legitimates the worrisome sentences rather than modifying or eliminating them, and it does so without requiring a significant change in our ontological commitments. While fictionalism thus seems to be an appealing approach, it has its own suite of difficulties to overcome. The most basic instrumentalist versions of fictionalism (such as the one proffered by Field) posit that we make-believe certain sentences are true in

(“Mathematical Metaphor” 31).

¹³⁰Jan Zwicky, “Mathematical Analogy and Metaphorical Insight,” *The Mathematical Intelligencer* 29.3 (2006): 9. It is important to note that, unlike the others, Zwicky does not argue that mathematics is fundamentally metaphorical. She even claims “No one is seriously going to maintain that mathematical analogies and metaphors are essentially the *same thing*,” though she believes they are relevantly similar and deeply connected (Zwicky, “Mathematical Analogy” 6).

¹³¹Several of Yablo’s papers address related issues, particularly “Does Ontology Rest on a Mistake?” (1998), “Abstract Objects: A Case Study” (2002), and “The Myth of the Seven” (2005). For clarity and simplicity, I have opted to focus on the treatment provided by Yablo in “Go Figure.”

¹³²Yablo, “Go Figure” 177.

¹³³Yablo, “Go Figure” 179.

order to serve some larger purpose, such as simplifying our theory; however, such positions stop short of providing an account of the mechanism of make-believing or the justificatory details underlying the fiction.¹³⁴ According to Yablo, this raises a variety of problems, including having no apparent way to distinguish correct and incorrect make-believe utterances.¹³⁵ Metafictionalism attempts to overcome this flaw of instrumentalist fictionalism by positing that by “making as if to assert [some utterance] S, one is really asserting that S is the right kind of thing to make as if to assert” within the game being played.¹³⁶ While this does provide a ground for distinguishing correct from incorrect make-believe utterances, it does not account for the apparent necessity of mathematical statements. It also leaves a problematic gap between mathematical fictions and the world that creates several puzzles surrounding the application of mathematical concepts: in metafictionalism, the underlying truths are *about* the game, not the world.¹³⁷

The first step to overcoming these issues is to recognize the difference between a sentence relying on rules and it being about those rules. Object fictionalism contends that a statement is *fictional* if an associated game of make-believe connects it to a real-world scenario which obtains (Yablo calls such a real-world scenario the *real content* of the associated fictional statement). For example, consider an Arthurian game in which a certain stick is designated as Excalibur. The statement “Darrell is wielding Excalibur” is fictional (“true-in-the-world-of-make-believe”) if Darrell is actually holding the designated stick.¹³⁸ Thus, in object fictionalism, the correctness of fictional statements relies upon the rules of a game of

¹³⁴For details on Field’s instrumentalist fictionalism, see his *Science Without Numbers* (1980) and *Realism, Mathematics, and Modality* (1989).

¹³⁵Yablo, “Go Figure” 179–80.

¹³⁶Yablo, “Go Figure” 181.

¹³⁷Yablo, “Go Figure” 181.

¹³⁸Walton, “Metaphor and Prop Oriented Make-Believe” 39. In Walton’s theory of make-believe, real-world objects and events that fictions depend on are called *props*. He compellingly argues that there may be an emphasis in games of make-believe which make them either content-oriented (where the prop is a means to the end of the fiction, a device which grounds the game) or prop-oriented (where the aim of the fiction is to facilitate efficient communication and/or improved understanding about the prop — for example, describing Italy as a boot). Note also that the designation that makes an ordinary stick into a prop of Excalibur requires an explicit declarative speech act in usual circumstances. It thus seems clear that theories of make-believe have a fundamental interest in the speech act of declaration insofar as it is a crucial mechanism for introducing new rules into a game. Hoffman provides a promising account of the role of speech acts in fiction in her dissertation “Mathematics as Make-Believe” (1999, pages 76–84), but focuses on directives to the exclusion of declarations.

make-believe, but is grounded in real-world circumstances; this development overcomes many of the problems associated with the previously mentioned versions of fictionalism. However, mathematical object fictionalism is itself susceptible to a powerful objection: it has no resources to handle sentences which enumerate collections of numbers. In object fictionalism, the statement “The number of continents on Earth is 7” is fictional because there are seven continents in the world but there is no number 7 in the world; that is, objects in the world directly ground the fictional statement. However, the statement “the number of even prime numbers is 1” is problematic for the object fictionalist because its real content would have to be that there is one even prime number in the world, a claim which no fictionalist can accept. Further, consider the statement “there are no numbers” — fictionalist philosophers of mathematics are prone to making this assertion. The associated fictional statement would be “the number of numbers is 0,” which is apparently self-refuting.¹³⁹

To get around this problem, Yablo draws a parallel between these apparently problematic statements and English statements of a similar form. Consider the following situation: Harry is a manure salesman who is delivering a tarp-covered trailer full of bovine excrement to Frank. However, Frank is erroneously convinced that Harry is lying to him and that there is no manure under the tarp. Attempting to persuade Frank that no swindling is taking place, Harry exclaims “This bullshit isn’t bullshit!” Read in a naive, context-independent way, this sentence appears to be self-refuting. However, when the first “bullshit” is taken literally and the second is taken figuratively, Harry’s statement is understood to be true.¹⁴⁰ Thus, Yablo suggests that, like the literal and figurative interpretations of the word “bullshit” in this example, number-talk can function in different ways that make sentences like those found in the previous paragraph unproblematic. Specifically, he claims that numbers can function as either representational aids (in referring to quantities, for example) or as things-represented (when talking *about* numbers), and that both functions sometimes occur within the same sentence (when talking about quantities of numbers, for example). Further, Yablo claims that such self-applied number-talk necessitates a mild relativism insofar as there are multiple legitimate interpretations of sentences such as “there are not many even primes.” That is,

¹³⁹Yablo, “Go Figure” 184.

¹⁴⁰Yablo, “Go Figure” 185.

the fictionalist can *engage* with the parasitic number game in their interpretation of this sentence when doing math (“there is only one even prime number, the rest are odd”) or *disengage* from it when talking about math (“there are no even numbers, as numbers do not exist”).¹⁴¹ Yablo refers to object fictionalism thus amended as *figuralism* to highlight the strong parallels between figurative language use and number-talk; the ubiquity of figurative utterances with multiple legitimate context-dependent interpretations is seen as vindicating the relativism of his approach.¹⁴²

Figuralism provides a systematic non-platonist answer to both its motivating puzzle and a variety of related questions about mathematical language use.¹⁴³ However, it is a controversial theory and has been criticized accordingly. The most noteworthy criticism, put forward by Burgess and Rosen, is a variation on their general argument against mathematical nominalism. The argument turns on the following distinction. The *hermeneutic* fictionalist contends that “*mathematicians’ own understanding* of their talk of mathematical entities is that it is a form of fiction, or akin to fiction: mathematics is like novels, fables, and so on in being a body of falsehoods *not intended to be taken for true*.”¹⁴⁴ The *revolutionary* fictionalist maintains that mathematicians understand their utterances to be literally true rather than fictional and are therefore systematically mistaken. Thus, hermeneutic fictionalists seek to provide a descriptive interpretation of mathematicians’ practices that reveals the lack of an ontological commitment to abstract mathematical entities, whereas the aims of revolutionary fictionalists are prescriptive, suggesting how to correct mathematics’ mistaken dependence on said commitment. The essence of Burgess and Rosen’s argument is that this distinction exhaustively partitions fictionalisms, and that both varieties are problem-

¹⁴¹Yablo, “Go Figure” 185. The idea of a parasitic game becomes necessary when considering engaged self-applied number-talk: whereas the fictionality of applied number-talk is directly grounded upon real-world circumstances, self-applied number fictions are indirectly grounded and depend on circumstances within applied number games for their correctness. In Waltonian terms, disengaged self-applied number talk is prop oriented as the props in the parasitic game are parts of the basic number game (Yablo, “The Myth of the Seven” 233–4). Note that “the appearance of stratification here is somewhat misleading, for parasitic games tend to swallow their hosts; instead of two games, one parasitic on the other, we wind up with a single game parasitic on itself” (Yablo, “Go Figure” 189). To me this seems analogous to how, in a sense, English serves as its own metalanguage.

¹⁴²Yablo, “Go Figure” 191.

¹⁴³Yablo, “Go Figure” 195–6.

¹⁴⁴John P. Burgess, “Mathematics and *Bleak House*,” *Philosophia Mathematica* 12.1 (2004): 23; emphasis his.

atic. Hermeneutic fictionalism fails because there is a distinct lack of empirical evidence that mathematicians interpret their findings as fictitious. Revolutionary fictionalism fails because there is overwhelming empirical evidence that mathematics works superbly as is, leaving us wondering why the purported mistakes require correction if they have no apparent impact on mathematics' success.¹⁴⁵

Given that Yablo says that figuralism was conceived in a "hermeneutic spirit," one might expect a defense specifically addressing the anti-hermeneutic horn of the argument.¹⁴⁶ However, it seems that the best defense strategies instead claim that the hermeneutic/revolutionary dilemma does not apply to figuralism in the way Burgess and Rosen suggest. One possible approach would be to deny that the distinction is exhaustive and claim that figuralism fits into some non-hermeneutic, non-revolutionary third category. This approach is potentially promising, but Burgess argues against it and I will not discuss it further here.¹⁴⁷ Sarah Hoffman argues figuralism has both hermeneutic and revolutionary aspects, and that these aspects are not problematic in the way Burgess and Rosen suggest. Yablo's view is only hermeneutic insofar as it is descriptive rather than prescriptive: he does not seek to alter mathematical practice. How, then, can figuralism be revolutionary if it is not prescriptive? The answer is that its prescriptions "do not seek to change mathematical practice or talk, but rather to depose the philosophical view that we *must* believe that there are mathematical objects in order to understand the way that (pure and applied) mathematics functions."¹⁴⁸ That is, Yablo's figuralist revolution would take place within the philosophy of mathematics, not within mathematics itself. Yablo's line of defense takes a slightly different approach than Hoffman's, though the two seem to cohere with each other. Yablo explicitly rejects nominalism because he feels its negative ontological conclusions (i.e., that mathematical objects do not exist) are stronger than warranted.¹⁴⁹ Therefore, insofar as Burgess and Rosen's argu-

¹⁴⁵Sarah Hoffman, "Yablo's Figuralist Account of Mathematics," 2012, Unpublished Article, 6.

¹⁴⁶Yablo, "Go Figure" 191.

¹⁴⁷Burgess 23n6.

¹⁴⁸Hoffman, "Yablo" 13.

¹⁴⁹Stephen Yablo, "Why *I* am Not a Nominalist," *Yablo's personal website* (2003). Yablo specifically requests that this work not be *quoted* as it is only an incomplete draft, but I have chosen to reference it because a) its title is very telling and b) it arguably provides the clearest presentation of certain claims that are consistent with commitments he makes elsewhere in publication. For example, in "Does Ontology Rest on a Mistake?" and "Must Existence-Questions Have Answers?" he contrasts both platonism and nominalism with an ontological stance that he calls *quizzicalism*: "Quizzicalists... find it hard to take (some? all?) ontological

ment targets nominalist theories, Yablo's figuralism seems immune to it. Though he denies that he is a nominalist, Yablo claims that he is nonetheless a hermeneutic fictionalist, arguing that the two viewpoints are distinct and that there is thus no inconsistency: hermeneutic fictionalism holds that mathematical utterances do not carry a commitment to numbers, while nominalism makes the stronger claim that numbers do not exist.¹⁵⁰

The difference between Yablo's and Burgess' characterizations of hermeneutic fictionalism points to a related criticism of figuralism that is more relevant to the present study: Burgess claims that Yablo's understanding of the literal/figurative distinction is problematic. In particular, Burgess claims that it is unlikely that mathematicians are speaking figuratively insofar as the non-literal use of language seems to require a conscious intention to diverge from the default literal usage; he thus places the burden of proof on Yablo to empirically confirm that such an intention indeed exists.¹⁵¹ Yablo counters not by providing proof that mathematicians intend to speak figuratively, but instead with psychological evidence that conscious recognition of non-literality is not a necessary component of successful figurative exchanges.¹⁵² The understanding of figurative language grounding Burgess' criticism seems to belong to the traditional linguistic approaches that the first half of this dissertation argued against, and thus Yablo's defense might seem decisive. On the other hand, Yablo's account of metaphor could benefit from some clarification. That any amount of clarification would be welcome is perhaps best illustrated by the following passage:

“say what you like about [the analytic/synthetic distinction], compared to the literal/metaphorical distinction it is a marvel of philosophical clarity and precision. Even those with use for the notion admit that the boundaries of the literal are about as blurry as they could be, the clear cases on either side enclosing a vast interior region of indeterminacy.”¹⁵³

What is clear is that Yablo's approach is based primarily upon Walton's theory of metaphor as prop oriented make-believe, and that he holds — along with many contemporary metaphor

debate seriously and hold out little hope for a successful resolution” (Yablo, “Must Existence-Questions Have Answers?” 297).

¹⁵⁰Yablo, “Why *I* am Not a Nominalist.”

¹⁵¹Burgess 26.

¹⁵²Yablo cites Gibbs here: “Figurative language interpretation does not follow after an obligatory literal misanalysis” (*The Poetics of Mind* 109).

¹⁵³Yablo, “Does Ontology Rest on a Mistake?” *Things* (New York: Oxford UP, 2010), 120.

theorists — that figurative language is neither rare nor exceptional; this provides a promising foundation for clarificatory efforts. One of the key results of the final section of this chapter is that CMT and embodied mathematics can help clarify and strengthen Yablonian figuralism, and vice versa.

4.4 Conceptual Metaphor Theory and the Philosophy of Mathematics

This chapter has surveyed a variety of mathematical viewpoints, each possessing distinctive strengths and weaknesses. All of the traditional viewpoints considered at the beginning of the chapter were ultimately rejected, and yet each seems to capture some important aspect of mathematics that traditional rival theories have difficulty accommodating. Unfortunately, the traditional viewpoints have usually been seen as strongly incompatible with one another, occasionally with tragic results.¹⁵⁴ Theories of mathematics typically do not exist in isolation but form part of a larger system of thought; these incompatibilities often derive from (or entail) more fundamental disagreements about the nature of truth or reality. Incompatible theories of truth traditionally leave little to no room for reconciliation within some overarching theory, largely due to their strictly antirealist commitments. One of the primary strengths of CMT is that because it is a general theory of concepts it functions as both a philosophy of mathematics as well as a philosophy of philosophies of mathematics.¹⁵⁵ In this latter role, CMT is promisingly inclusionary: conceptual metaphor provides a mechanism for integrating the strongest aspects of various *inconsistent* theories of mathematics into a *coherent* whole. This final section of this chapter sketches compatibilities between embodied mathematics and other mathematical theories, strengthening the explanatory power of the theory and

¹⁵⁴Kronecker's intuitionist objections to Cantor's transfinite arguments famously generated professional conflict that prevented Cantor from obtaining a professorship worthy of his talent, and ultimately contributed to the mental health issues that plagued him in his later years (Barrow 198–204). A similar (though somewhat less tragic) public rivalry between the intuitionist Brouwer and the formalist Hilbert was dubbed the *Frosch-Mäusekrieg* (or War of the Frogs and the Mice) by Einstein (Barrow 216–26).

¹⁵⁵Part III of Lakoff and Johnson's *Philosophy in the Flesh* considers various major philosophical movements throughout history from the CMT viewpoint, describing the predominant conceptual metaphors underlying each.

suggesting directions for future research and development.

The claim that CMT can form the basis of an integrated philosophy of mathematics may seem immediately false given that Lakoff and Núñez say that “*Mind-based mathematics*, as we describe it in this book, is not consistent with any of the existing philosophies of mathematics: Platonism, intuitionism, and formalism. Nor is it consistent with recent post-modernist accounts of mathematics as a purely social construction.”¹⁵⁶ Additionally, Lakoff and Núñez take particular issue with the platonistic “Romance of Mathematics,” arguing that conceiving of mathematics as transcendental rather than constitutively embodied is not merely empirically unsubstantiated but harmless, but rather that dehumanizing and elevating mathematics does social harm by making the subject seem arcane and frustratingly inaccessible to many students.¹⁵⁷ Despite Lakoff and Núñez’s vehement opposition to full-fledged platonism, embodied mathematics allows for — and, indeed, even requires — certain aspects of mathematical platonism; if embodied mathematics can successfully incorporate elements of its staunchest rival theory, it is plausible that it can integrate aspects of other theories as well.

It is worth noting that Lakoff and Núñez do not definitively reject the possibility of the existence of a transcendental mathematics. What they do argue is that there is no (and can be no) evidence establishing the existence of transcendental mathematical objects, and that even if they were to exist, our embodied mathematical concepts would be independent from them, not representations of them.¹⁵⁸ This kind of compatibility is so minimal it is hardly worth mentioning, but there is a more robust sense in which platonism and embodied mathematics are compatible. Lakoff and Núñez acknowledge that we do, in part, conceptualize numbers as objects, and have no desire to eliminate such talk. They claim that the four grounding metaphors of arithmetic together naturally induce the more general conceptual metaphor NUMBERS ARE THINGS IN THE WORLD.¹⁵⁹ Their brief discussion asserts that this metaphor is an integral part of our understanding of mathematics, allowing for efficient communication and providing structure for important notions such as mathematical closure by way of its

¹⁵⁶ *Where Mathematics Comes From* 9; emphasis theirs.

¹⁵⁷ *Where Mathematics Comes From* 341.

¹⁵⁸ Lakoff and Núñez, *Where Mathematics Comes From* 4.

¹⁵⁹ Lakoff and Núñez, *Where Mathematics Comes From* 80.

inference-preserving nature.¹⁶⁰ While Lakoff and Núñez see NUMBERS ARE THINGS IN THE WORLD as a natural and useful part of mathematics, at the level of the philosophy of mathematics, they wish to acknowledge its metaphoricity and therefore disallow its use in making stronger metaphysical inferences. In this, there seem to be important similarities between the Lakovian and Yablonian approaches to mathematical object talk, connexions that will be discussed shortly.

As was the case with platonism, embodied mathematics embraces the mathematical practices associated with formalism while simultaneously acknowledging its metaphoricity and rejecting its philosophical pretensions.¹⁶¹ That is, Lakoff and Núñez endorse the idea that formal symbolization and the axiomatic method are an important aspect of mathematics, but reject the idea that mathematics is reducible to such an approach: “Mathematics is not literally reducible to set theory in a way that preserves conceptual differences. However, ingenious metaphors linking ordered pairs to sets and numbers to sets have been explicitly constructed and give rise to interesting mathematics.”¹⁶² They claim that there is a metaphorical schema — dubbed the FORMAL REDUCTION METAPHOR — which allows all mathematical notions to be conceptualized in terms of axiomatic set theory: “the Formal Reduction Metaphor inherently makes an interesting claim: The relatively impoverished conceptual structure of set theory and formal logic is sufficient to characterize the structure (though not the cognitive content) of every mathematical proof in every branch of mathematics.”¹⁶³ The FORMAL REDUCTION METAPHOR is a collection of linking metaphors that provide unifying structure to mathematics, connecting disparate branches through a set theoretic hub.¹⁶⁴ Traditional formalists see

¹⁶⁰Lakoff and Núñez, *Where Mathematics Comes From* 81. Performing a mathematical operation on elements of a given set sometimes yields a result that is not contained in the set, such as when one subtracts a larger natural number from a smaller one. Informally, the idea of closure is that such a set can often be enriched with further elements so that the result of performing the operation on the elements of the enriched set always lands within the enriched set. Pimm (1981) also discusses the role of metaphor in closure.

¹⁶¹As was discussed at the beginning of the chapter, the boundary between logicism and formalism becomes blurred when one focuses on mathematical practice without considering the philosophical underpinnings. It seems likely that this is one of the main reasons Lakoff and Núñez only use the word “logicism” once, in passing, in *Where Mathematics Comes From*. Following their lead, only formalism will be discussed in this paragraph, but the reader should understand the discussion to pertain to logicism as well.

¹⁶²Lakoff and Núñez, *Where Mathematics Comes From* 152.

¹⁶³Lakoff and Núñez, *Where Mathematics Comes From* 373.

¹⁶⁴Whether it constitutes part of the FORMAL REDUCTION METAPHOR or is merely associated with it, most explanations of formalism invoke the metaphor MATHEMATICS IS A GAME.

these connexions as presenting an opportunity to reduce mathematics to set theory, thereby avoiding multiplication of entities. Lakoff and Núñez argue that it is desirable to view these connexions as allowing for increased richness of structure in mathematics through conceptual blending rather than to use them to eliminate other branches of mathematics through reduction, which would lead to a starker, more restrictive mathematics; part of the evidence that they present in favour of their interpretation is that the experimental data conflicts with reductivist formalism. Additionally, formalistic reduction has difficulties taking into account the various versions of formal logic and set theory.¹⁶⁵ There are more dimensions to our mathematical conceptualizations than set theory alone affords, and the fascinating and powerful capacity of mathematics to cross-model itself should be embraced and utilized, not invoked self-defeatingly.¹⁶⁶

While embodied mathematics incorporates the mathematical practices of the platonist and the formalist while rejecting their philosophical claims, the opposite holds true in its relationship with most constructivists. Like constructivism, embodied realism posits that mathematics is a construct of the human mind: “If there are ‘foundations’ for mathematics, they are *conceptual foundations* — *mind-based foundations*.”¹⁶⁷ Lakoff and Núñez’s account of how limited inborn numerical capacities such as subitization give rise to the full panoply of quantitative mathematics brings to mind Kronecker’s famous dictum: “The integers were made by God; all else is the work of man.”¹⁶⁸ However, embodied mathematics rejects the intuitionist restriction of mathematical practice to constructive methods as unwarranted and needlessly limiting. Lakoff and Núñez claim that the law of the excluded middle — generally disallowed to some extent in intuitionist mathematics — emerges naturally in our understanding of categories, transferred via conceptual metaphor from the spatial logic inherent in the container schema.¹⁶⁹ Whereas intuitionists do not admit infinity into their mathemati-

¹⁶⁵ *Where Mathematics Comes From* 374.

¹⁶⁶ Proponents of embodied mathematics hold that logic depends on the embodied mind, and thus there is a neurological conceptual system behind all axiomatics. Applying Occam’s razor from within the logical domain to cut off all non-set-theoretic aspects of our mathematical concepts seems to constitute a kind of category mistake, unless one assumes that all reasoning and conceptualization is axiomatic logic all the way down. However, this begs the question.

¹⁶⁷ Lakoff and Núñez, *Where Mathematics Comes From* 376; emphasis theirs.

¹⁶⁸ Barrow 188.

¹⁶⁹ *Where Mathematics Comes From* 44.

cal practices (as finite humans cannot literally construct infinite objects), using the BMI to explain how humans metaphorically create infinities is one of the main objectives of *Where Mathematics Comes From*.¹⁷⁰ Thus, while Lakoff and Núñez and the intuitionists agree that mathematics is a human construct, they differ significantly on what is being constructed and how.¹⁷¹

Embodied mathematics is consistent with weak social constructivism insofar as it acknowledges that cultural factors and historical accidents play an important role in the development of mathematics:

Many of the most important ideas in mathematics have come not out of mathematics itself, but arise from more general aspects of culture. The reason is obvious. Mathematics always occurs in a cultural setting. General cultural worldviews will naturally apply to mathematics as a special case. In some cases, the result will be a major change in the content of mathematics itself.¹⁷²

However, unlike more radical social constructivist theories, embodied mathematics “*explicitly rejects any possible claim that mathematics is arbitrarily shaped by history and culture alone.*”¹⁷³ Radical social constructivism tends to be associated with problematic “anything goes” relativism. Accordingly, such theories have inadequate resources to account for the perceived nonarbitrariness of mathematics. Lakoff and Núñez, on the other hand, hold that culture and history are merely two among many factors contributing to mathematical development, and it is embodiment that prevents mathematics from being arbitrary. As they put it,

Where does mathematics come from? It comes from us! We create it, but it is not arbitrary—not a mere historically contingent social construction. What makes mathematics nonarbitrary is that it uses the basic conceptual mechanisms of the embodied human mind as it has evolved in the real world. Mathematics is a product of the neural capacities of our brains, the nature of our bodies, our evolution, our environment, and our long social and cultural history.¹⁷⁴

¹⁷⁰I realize that there are various strains of intuitionism, and many of them are not strictly finitistic; however, even those intuitionisms that do admit some idea of infinity will have a limited understanding compared to the non-intuitionist.

¹⁷¹Though Lakoff and Núñez explicitly tell us that embodied mathematics is inconsistent with intuitionism, this is the only time they use the word in their book; they never elaborate on the inconsistency (*Where Mathematics Comes From* 9).

¹⁷²Lakoff and Núñez, *Where Mathematics Comes From* 358.

¹⁷³Lakoff and Núñez, *Where Mathematics Comes From* 362; emphasis theirs.

¹⁷⁴Lakoff and Núñez, *Where Mathematics Comes From* 9.

Cultural and historical factors may be of peripheral importance compared to embodiment in the Lakovian theory, but it is worth looking a little closer at the contribution they make.

For Lakoff and Núñez, embodied experience accounts for the stability, precision, consistency, discoverability, and universality of mathematics.¹⁷⁵ Thus, if culture has a distinctive role to play, it will involve the non-universal aspects of mathematical concepts. This is consistent with CMT in general, as per the explanation in chapter 3: while primary metaphors tend to be ubiquitous due to their direct grounding, there is significantly more room for cultural variation in the complex metaphors that structure our abstract concepts. A considerable amount of anthropological and sociological research has sought to find and explain noteworthy conceptual differences between the mathematics of isolated peoples and our own; recall, for example, the finding that the Yupnos of Papua New Guinea lack the linking metaphor NUMBERS ARE POINTS ON A LINE. The relationship between culture and concept seems complicated and bidirectional: not only do different cultures develop different concepts due to different shared experiences, but some cultures may originate from the shared conceptual commitments of their members. This seems particularly relevant in mathematics, where a formalist may choose which axioms to adopt: a community of mathematicians who *opt* to practice an alternative form of mathematics (non-Euclidean geometry, non-well-founded set theory, etc.) may constitute a culture. Thus, for Lakoff and Núñez the very *possibility* of inconsistent but equally legitimate coexisting alternative forms of mathematics exposes mathematics' cultural dimension.

Of course, such mathematical differences can occur vertically as well as laterally; that is, mathematics exhibits both diachronic and synchronic variation. The apparently timeless nature of mathematics has a tendency to obfuscate the history of mathematical concepts even more than happens in other disciplines. If, for some reason, one does think about the history of mathematics, thanks to the traditional dominance of platonism and formalism there is a strong temptation to think of it as a gradual expansion of knowledge, analogous to either the discovery and conquest of new, uninhabited territories or the meticulous piece-by-piece assembly of a structure unfolding from a firm foundation into unoccupied space, or some blend of the two. A key point of disanalogy between these two viewpoints and the history

¹⁷⁵ *Where Mathematics Comes From* 350.

of mathematics comes in with the words “uninhabited” and “unoccupied”: mathematical progress does not occur in a vacuum but rather is constrained and guided by accidental elements such as puzzles and problems arising in other areas, technological advances, political climate, etc.¹⁷⁶ These constraints are often insufficient to uniquely determine the expansion of mathematical knowledge, forcing mathematicians to make creative choices that influence the way the discipline progresses. To illustrate, Lakoff and Núñez briefly discuss some ways that the advent of the computer helped shape mathematics. On the one hand, computer technology eventually facilitated computations and visualizations that were not previously feasible, allowing for the development of fractal geometry, for example. On the other hand, the digital nature of computer technology is not immediately well suited to mathematics involving the continuum of real numbers. To improve the way computers approximate real numbers and enhance the applicability of the technology, mathematicians chose to develop new floating-point arithmetics. These arithmetics possess some features that would seem very unusual to most people — such as considering positive zero and negative zero to be distinct numbers — and yet, given the ubiquity of computers and the incredible rapidity of their computations, the vast majority of arithmetic takes place within a floating-point context.¹⁷⁷

While Lakoff and Núñez acknowledge the historical dimension of mathematics, their mathematical idea analysis is predominantly synchronic; incorporating a more-thorough account of the diachronics of mathematics is desirable. Establishing coherences between the Lakovian theory and Imre Lakatos’ quasi-empiricist account of mathematics would go a considerable distance towards attaining a balanced understanding of mathematical conceptual development. It is not possible to fully integrate the two theories here, but a groundwork for synthesis can be sketched. In *Proofs and Refutations* — Lakatos’ primary work in the phi-

¹⁷⁶Even taboos can have a significant impact on the way mathematics unfolds. Two important examples spring immediately to mind. The Pythagoreans believed that the entire universe could be explained in terms of ratios of whole numbers, a commitment that made thought about irrational quantities blasphemous; those who questioned the ultimate status of the rational numbers were allegedly exiled or put to death. Eventually, demonstration of the irrationality of $\sqrt{2}$ was instrumental in the dissolution of the cult (Kline 32–3). The Muslim proscription against idolatry meant that abstract geometric designs were often employed in decorating their architecture, facilitating a productive, bidirectional connexion between decorative art and mathematical theory (*Islamic Art and Geometry* 10).

¹⁷⁷Lakoff and Núñez, *Where Mathematics Comes From*, 360–1.

losophy of mathematics — he presents a picture of mathematical progress that is far more complicated and tumultuous than the slick definitions and proofs that are the end result of that process suggest. The majority of the book presents a beautiful case study in dialogue format: a teacher and a classroom full of pupils discuss how to go about proving Euler’s Polyhedron Theorem (the formulation which serves as the starting point of the dialogue: for all polyhedra, Vertices + Faces - Edges = 2).¹⁷⁸ This result is easily confirmed for specific polyhedra — such as the Platonic and Archimedean solids — but such confirmations do not constitute a proof, and thus the formula is not yet a theorem but only an inductively motivated conjecture.¹⁷⁹ Lakatos’ dialogue follows the historical record, glossing many decades of mathematical debate. A proof (Cauchy’s, in this particular case) is offered, lending substance to the formula but also providing opportunity and inspiration for the generation of counterexamples based upon the specific assumptions invoked and methods utilized. Any such counterexamples which arise invite a response that tends to fall somewhere between the extremes of repudiation and capitulation, some qualification or clarification of the concepts and techniques of the proof that takes the counter out of the examples. The improvements made to the proof may provoke further counterexamples, resulting in a cyclic process of conceptual negotiation and refinement. The details of this process of negotiation are the principal focus of Lakatos’ book; unfortunately, a comprehensive discussion of his insights is not possible here. Lakatos’ understanding of mathematical development is a blend of Popperian falsificationism and Hegelian thesis-antithesis-synthesis dialectic; mathematics, then, is quasi-empirical insofar as it is a *practice* that proceeds like the physical sciences, only with thought-experiments in place of laboratory work.¹⁸⁰ Well-defined mathematical concepts only emerge as the process of proofs and refutations reaches an equilibrium; thus, the often “artificial and mystifyingly complicated” looking axioms and definitions are the end result of mathematical practice and not the *ex nihilo* origin, as the clean axiomatic deductions that

¹⁷⁸Imre Lakatos, *Proofs and Refutations* (New York: Cambridge UP, 1976), 7.

¹⁷⁹The Platonic solids (tetrahedron, cube, octahedron, dodecahedron, icosahedron) are familiar to most, certainly to those who have ever played a role-playing game like Dungeons and Dragons requiring a variety of non-standard dice (Sutton 2). The Archimedean solids are less commonly known: they are semi-regular convex polyhedra with two (or more) distinct types of regular polygonal faces. There are 13 of them (15 if chiral pairs are distinguished) (Sutton 32, 44).

¹⁸⁰Lakatos 9, 144–5, 149.

pave over the dynamic spadework suggest.¹⁸¹ Like Lakoff and Núñez, Lakatos warns against attempting to ontologically reduce mathematics to the formal.¹⁸²

The suggestion that mutually beneficial coherences exist between Lakoff and Lakatos is not unprecedented: in *Women, Fire, and Dangerous Things*, Lakoff cites Lakatos in his short list of noteworthy scholars involved in overturning the classical theory of concepts.¹⁸³ Both theories hold that concepts are not necessarily essentially definitional, and emphasize non-formalistic mathematical practice as fundamental. What Lakatos offers is a general account of the dynamics of mathematical conceptual development, invoking the metaphor MATHEMATICAL HISTORY IS A DIALOGUE in the telling. Lakatos' account of the concept POLYHEDRON seems compatible with Lakoff's theory of categorization: while certain polyhedra (such as cubes and the other Platonic solids) are prototypical of the class, the counterexamples which arise force mathematicians to make precise the radial structure of the category; thus understood, the structure of POLYHEDRON and MOTHER appear similar, though the histories of the two concepts are surely radically different.¹⁸⁴ Thus, Lakatos provides an account of the development of mathematical concepts drawn from historical evidence that is coherent with CMT and therefore may augment the diachronic dimension of that theory. On the other hand, a Lakatosian could use this coherence with Lakovian embodied realism to help counter charges of relativism. Exploring the depth of this coherence and the fruitful syntheses it engenders is a worthy task. This discussion must wait for another time, as there are other important coherences to consider.

There are obvious connexions between metaphor and fiction. Both “metaphorical” and “fictional” are used as antonyms for “literal,” a connexion backed up by the related common belief that, while accurate literal statements are true, fictions and metaphors are falsehoods.¹⁸⁵ Though skillful critical thinking practice cautions against relying on negative definitions, this shared opposition indicates the two ideas are related to some degree. It is

¹⁸¹Lakatos 142.

¹⁸²David Bloor, *Knowledge and Social Imagery*, 2nd ed. (Chicago: U of Chicago P, 1991), 152.

¹⁸³*Women, Fire, and Dangerous Things* 17.

¹⁸⁴Lakoff, *Women, Fire, and Dangerous Things* 83.

¹⁸⁵Recall that conceptual metaphor theorists reject the literal/metaphorical opposition, at least to the extent that statements understood as obviously and directly true frequently invoke conceptual metaphors (Lakoff and Johnson, *Metaphors We Live By* 5).

also clear that fiction and metaphor can and do interact with each other. Metaphors frequently occur in works of literary fiction: “What light through yonder window breaks? / It is the east, and Juliet is the sun!”, one of the most frequently cited examples of linguistic metaphor, is invoked by Shakespeare to convey fictional Romeo’s feelings and intentions towards fictional Juliet to the reader.¹⁸⁶ Conversely, fictional entities and situations can figure in linguistic metaphors, as in Searle’s example “Sally is a dragon.”¹⁸⁷ Further, an extended passage or even an entire fictional work can be read allegorically, used as a metaphorical device for better understanding something apart from the fiction, regardless of the author’s intentions. Finally, some philosophical work has been done towards establishing deeper connexions between metaphor and fiction, typically focused on trying to explain the former in terms of the latter. For example, Kendall Walton argues that it is advantageous to understand at least some metaphors in terms of prop oriented make-believe.¹⁸⁸ Many of Walton’s writings are dedicated to developing a theory of fiction rooted in imagination, pretense, and games of make-believe. Games of make-believe typically rely on props, “generators of fictional truths” which provide structure to the fiction; props considered by Walton range from simple children’s toys to literary fiction and other representational artworks.¹⁸⁹ The majority of Walton’s writing focuses on *content oriented* make-believe, where the focus is on the fictional world of the game and associated props serve as a means to that end. However, he does also give some attention to discussing *prop oriented* make-believe, where “interest is focussed on the props themselves; the envisioned make-believe provides a way of describing them.”¹⁹⁰ Stage props featured in a theatrical production are an example of the former, while referring to a person wearing spectacles as “four eyes” is an instance of the latter. Walton claims that many metaphorical statements invoke a game of prop oriented make-believe; for example, when a plumber refers to a certain fixture as “male” it is likely that she is doing so merely as a referential convenience and that her focus is on the protruding (as opposed to recessed) nature of the fixture rather than exploring a game where inanimate plumbing parts

¹⁸⁶William Shakespeare, *Romeo and Juliet* II.ii.

¹⁸⁷“Metaphor” 87.

¹⁸⁸“Metaphor and Prop Oriented Make-Believe” 45.

¹⁸⁹Kendall Walton, *Mimesis as Make-Believe* (Cambridge: Harvard UP, 1990), 37–8.

¹⁹⁰Walton, “Metaphor and Prop Oriented Make-Believe” 43.

are gendered.¹⁹¹ On the other hand, more elaborate, open-ended metaphors may “function something like the stipulative launching of a (content oriented) game of make-believe, which then grows naturally beyond the original stipulation.”¹⁹² Some metaphors may transition between the two orientations; for example, this would be the case if the aforementioned plumber began extending and enriching her fixture metaphor beyond male/female to include hermaphrodites, eunuchs, the transgendered, and so forth. Though he is the first to admit that he is not proposing a *theory* of metaphor, Walton’s observations provide an interesting and useful way of thinking about metaphors.¹⁹³

Considering these connexions, it is somewhat surprising that nobody has looked at integrating Lakovian embodied mathematics with mathematical fictionalism before now. All versions of fictionalism draw an analogy between mathematics and fiction to help reconcile our mathematical practices with a commitment to the non-existence of mathematical entities.¹⁹⁴ In terms of CMT, mathematical fictionalism fundamentally depends on the metaphor MATHEMATICS IS FICTION. Interestingly, unlike platonists, most fictionalists acknowledge that a metaphor underlies their viewpoint and that there are important disanalogies between mathematics and fiction. For people suspicious about the existence of abstract entities, there seems to be a natural progression from using the platonist metaphor NUMBERS ARE THINGS IN THE WORLD while doing mathematics, to recognizing it as a metaphor when thinking about mathematics, to turning to the metaphor MATHEMATICS IS FICTION to help in making sense of the metaphoricity of NUMBERS ARE THINGS IN THE WORLD; that is, recognizing that numbers are not things in the world like frogs and toasters raises a suite of questions about what is going on when sentences seem to refer to numbers, questions that MATHEMATICS IS FICTION can help define. Thus, mathematical fictionalism can be understood in terms of CMT, and can enhance embodied mathematics by providing resources to help understand how mathematical

¹⁹¹ “Metaphor and Prop Oriented Make-Believe 40.

¹⁹² Walton, “Metaphor and Prop Oriented Make-Believe 53.

¹⁹³ Walton, “Metaphor and Prop Oriented Make-Believe” 45. It is worth noting that Elisabeth Camp argues that “metaphors generally involve a different sort of imaginative game than make-believe [“seeing-as”], and that utterances which do evoke make believe are not usually metaphors” (“Two Varieties of Literary Imagination” 110). Her arguments that pretense and seeing-as are distinct, important activities of the imagination are compelling and certainly warrant further consideration. However, as my viewpoint does not depend on reducing metaphor to fiction or fiction to metaphor, I will not discuss her position further here.

¹⁹⁴ Mark Balaguer, “Fictionalism in the Philosophy of Mathematics,” *Stanford Encyclopedia of Philosophy*, 30 Jul. 2012.

sentences refer and what they are referring to.

Building on the above, the connexions and coherences between Lakovian embodied mathematics and Yablonian figuralism suggest that a successful partnership might be forged between the two theories. Figuralism differs from generic fictionalism in a few important ways that makes it a better fit for embodied mathematics; these differences lead some to question whether Yablo's theory should be classified as a variety of fictionalism.¹⁹⁵ Most fictionalists assert that mathematical statements are false because they involve failed direct reference to non-existent mathematical objects. Yablo says that mathematical statements can be true because they may refer *indirectly* to an underlying real state of affairs by way of a game of make-believe: "one gives voice to the real truth by making as if to assert the fictional truth that it enables."¹⁹⁶ Thus, at the heart of Yablo's view resides Walton's prop oriented make-believe applied to the case of mathematical utterances. Walton, then, is one connexion between Yablo and Lakoff: in "Metaphor and Prop Oriented Make-Believe," he cites Lakoff and Johnson and uses ARGUMENT IS WAR as an explicit example of prop oriented make-believe, providing a brief analysis that seems to cohere with CMT.¹⁹⁷ This is not the only connexion between the two theories. Both viewpoints are mildly relativist: Yablo presents the term *relative* reflexive fictionalism as practically synonymous with figuralism, and notes that distinguishing engaged and disengaged modes of understanding involves "a kind of relativism".¹⁹⁸ Both viewpoints are practice-oriented, taking the empirical observation that our mathematical practices are successful as fundamental. Indeed, both Lakoff and Yablo concede that, despite doubts about the existence of mathematical objects, it is possible that they could have an objective existence beyond our ken; however, precisely because such objects would be beyond our experience, their ultimate ontological status must be irrelevant to mathematics as we know it.¹⁹⁹ Yablo's description of object fictionalism's Waltonian core makes heavy use of the metaphor FICTIONAL STATEMENTS ARE EXPONENTIAL FUNCTIONS, a move that conceptual metaphor theorists can both understand and appreciate.²⁰⁰ And, of course, last but not least,

¹⁹⁵Balaguer, "Fictionalism in the Philosophy of Mathematics."

¹⁹⁶"Go Figure" 183.

¹⁹⁷"Metaphor and Prop Oriented Make-Believe" 45.

¹⁹⁸"Go Figure" 189.

¹⁹⁹Yablo, "Go Figure" 193; Lakoff and Núñez, *Where Mathematics Comes From* 342–3.

²⁰⁰"Go Figure" 182.

both theories understand mathematical practice as fundamentally involving metaphor.

So the two theories seem to have enough in common to play well together. What motivation is there for attempting to integrate them? That is, what can Lakoff and Yablo provide each other? Despite the various commonalities discussed above, figuralism and embodied mathematics come at explaining mathematical practice from different directions. Embodied mathematics works from the inside out, sketching how our vast system of mathematical concepts emerges from basic biological capacities and common experiences. Figuralism works from the outside in, focusing on how mathematical language functions. Thus, the two viewpoints have different strengths that can complement each other, helping shore up weaknesses. It was observed in chapter 3 that Lakoff's focus on conceptual metaphor means that he neglects to spend much time considering the details of how metaphorical utterances relate to concepts. Additionally, nowhere does Lakoff provide an account of fiction based in CMT. Yablo's Waltonian approach and its core metaphor provide a foothold for enriching this aspect of the Lakoffian theory, particularly his understanding of reference. Conceptual metaphors and games of make-believe are posited to play the same role underpinning metaphorical utterances; the relationship between these two cognitive devices requires investigation.²⁰¹ Yablo's clever handling of "the bomb" of self-applied number talk also provides CMT with resources that could be used to fend off worries about abstract objects and circularity associated with the concept of CONCEPT; while numerical mathematical sentences are his main focus, it is worth noting that Yablo's discussion aims to handle sentences purporting to refer to abstract entities in general. On the other hand, Yablo's focus on the linguistic side of things is certainly a contributing factor in his bemoaning the lack of clarity in the literal/metaphorical distinction; understanding metaphor as conceptual can help clarify this situation. Finally, wedding Yablo to Lakoff would be beneficial for the latter insofar as the relationship could bring increased philosophical rigour and respect: Yablo is a respected and prolific member of the contemporary philosophical community and, moreover, the idea that fiction is involved in mathematics has been endorsed by many philosophers, including Bertrand Russell.²⁰² Much

²⁰¹ Elisabeth Camp's arguments in "Two Varieties of Literary Imagination" distinguishing pretense from seeing-as should be taken into account in any such future study.

²⁰² "If there has been any truth in the doctrines that we have been considering, all numbers are what I call logical fictions. Numbers are classes of classes, and classes are logical fictions, so that numbers are, as it

more work would be required to actually synthesize these two views in a fruitful way, and there would likely be significant resistance from both sides; regardless, the potential gains to be had from combining the Lakovian and Yablonian approaches makes further consideration of this merger desirable.

At the end of chapter 3, it was suggested that incorporating semiotic insights could help CMT overcome some of its most serious deficiencies. If semiotics has the potential to bring balance to CMT generally, then it seems reasonable that it could make a beneficial contribution to the special case of embodied mathematics as well. Brian Rotman has made a career of “giving a semiotic analysis of mathematical signs,” and his work is not only coherent with embodied mathematics but explicitly connects itself to many of the other mathematical theories that have been considered in this chapter.²⁰³ His inquiry starts in a way nearly identical to the one presented in this chapter: it explicitly understands mathematics as a practice, and expresses dissatisfaction with the big three traditional theories of mathematics while acknowledging that

to have persisted so long each must encapsulate, however partially, an important facet of what is felt to be intrinsic to mathematical activity. Certainly, in some undeniable but obscure way, mathematics seems at the same time to be a meaningless game, a subjective construction, and a source of objective truth. The difficulty is to extract these part-truths: the three accounts seem locked in an impasse which cannot be escaped from within the common terms that have allowed them to impinge on each other.²⁰⁴

Observing that mathematical writing is permeated with imperatives — exhortations to “consider,” “define,” “add,” “integrate,” and so forth — Rotman asks what kind of actions result from such requests given that “mathematics can be an activity whose practice is silent and sedentary.”²⁰⁵ Drawing inspiration from Peirce, he concludes that the mathematical listener

were, fictions at two removes, fictions of fictions” (Russell, *The Philosophy of Logical Atomism* 111).

²⁰³Brian Rotman, *Mathematics as Sign: Writing, Imagining, Counting* (Stanford: Stanford UP, 2000), 4. It is somewhat curious that relatively little work has been done in the semiotics of mathematics given the prominent role of signs in that discipline. Moreover, the idea of applying semiotics to math is hardly new. Over a century before Rotman, Peirce made multiple references to mathematics in his work on signs (Rotman, *Mathematics as Sign* 4). Even earlier, Bolzano discussed the semiotics of mathematical symbols. Interestingly, Bolzano foreshadows Lakatos, claiming that purposes precede definitions and “therefore it is an error, contrary to good method, when Euclid gathers all his definitions at the beginning” (Bolzano 107).

²⁰⁴Rotman, *Mathematics as Sign* 7.

²⁰⁵Rotman, *Mathematics as Sign* 12. As mentioned in a previous note, Hoffman also emphasizes the role of imperatives in mathematical practice in her dissertation.

responds to an imperative by engaging in an imaginative kind of thinking akin to a thought experiment that is often inseparably amalgamated with “scribbling.” To understand these interactions in a way coherent with the insights of the traditional theories, Rotman conceives the agency of the mathematician as a trinity: Agent, Subject, Person. The Agent is an automaton, a minimal, imaginary avatar of the mathematician that follows rules to perform mathematical tasks; devoid of subjectivity, the Agent transcends finitude and other logical constraints. The Subject is the master of the Agent, the authorial agency whose imaginings form the worlds that the Agent is deployed into. As Rotman puts it, “our picture of the Subject is of a conscious — intentional, imagining — subject who creates a fictional self, the Agent, and fictional worlds within which this self acts. But such creation cannot, of course, be effected as pure thinking: signifieds are inseparable from signifiers: in order to create fictions, the Subject scribbles.”²⁰⁶ While the Subject is considerably more robust (though less transcendental) than the Agent, her psychology is unsituated (“transcultural and disembodied,” in Rotman’s words) and entirely engaged with mathematics.²⁰⁷ The Person is the embodied agency of the mathematician, capable of using natural language, constrained and motivated by history, culture, and the practicalities of the world: the situated, unprojected self.²⁰⁸ This conception of mathematical agency does a lot of work for Rotman, including allowing for well-structured discussion of where each of the traditional theories goes wrong. Note that, in terms of this trifurcation, both Lakoff and Lakatos believe that other theories have problematically neglected consideration of the contributions of the Person to mathematics.²⁰⁹ Though this presentation of Rotman’s view is brief and incomplete, it is sufficient for

²⁰⁶ *Mathematics as Sign* 14.

²⁰⁷ *Mathematics as Sign* 15.

²⁰⁸ Rotman, *Mathematics as Sign* 15. Lakatos seems to endorse the Subject-Person distinction in the following passage:

mathematical activity produces mathematics. Mathematics, this product of human activity, ‘alienates itself’ from the human activity which has been producing it. It becomes a living, growing organism that *acquires a certain autonomy* from the activity which has produced it; it develops its own autonomous laws of growth, its own dialectic. The genuine creative mathematician is just a personification, an incarnation of these laws which can only realise themselves in human action (146).

²⁰⁹ Rotman mirrors a major Lakatosian theme when he says “in the absence of the Person’s role, no explication of conviction — without which proofs are not proofs — can be given.” (*Mathematics as Sign* 54).

the purpose of briefly considering the coherences between his semiotic account and embodied mathematics.

It is clear from the previous paragraph that, like Lakoff, Rotman focuses on mathematics as practice. His approach is also similar to CMT in its inclusionary attitude: while Rotman believes that the three traditional theories are incompatible and so a synoptic reconciliation is impossible, he makes a concerted effort to understand each of them from the context of his semiotic theory and incorporate their strengths where possible. He acknowledges that metaphor seems to play an ineliminable role in mathematics, and also draws a connexion to fictionalism in this significant passage:

any attempt to explicate mathematical thought is unlikely to escape the net of ... metaphors; indeed, to speak (as we did) of dwelling in a world of Hausdorff spaces is metaphorically to equate mathematical thinking with physical exploration. Clearly, such worlds are imagined, and the actions that take place within these worlds are imagined actions.²¹⁰

Though they seem to disagree slightly about the details of the contribution, Rotman and Lakoff and Núñez agree that gesture plays an important role in mathematical practice and development.²¹¹ An intriguing but more tenuous connexion between CMT and Rotman comes from Zoltán Kövecses, one of Lakoff's collaborators. He distinguishes three levels on which conceptual metaphors can be analyzed (the supraindividual, the individual, and the subindividual) which seem to roughly correspond with Rotman's trinity of mathematical agency insofar as they involve similar restrictions of scope.²¹² These coherences with Rotman offer Lakoff a significant first step towards integrating semiotic insights into embodied mathematics, hopefully bringing balance to the theory by distributing focus across the semiotic triangle rather than dwelling on the conceptual. Additionally, Rotman brings his own collection of useful coherences with various mathematical theories that could help strengthen those of Lakoff. It would be particularly interesting to compare and contrast the Subject-Agent account of infinite operations with the Basic Metaphor of Infinity, exploring the interplay and synergies between these two approaches.²¹³ While it would be great to incorporate semiotic

²¹⁰Rotman, *Mathematics as Sign* 12–3.

²¹¹Brian Rotman, "Gesture in the Head: Mathematics and Mobility," *Mathematics and Narrative conference, Mykonos, 2005*, 15–6. See also Núñez (2008).

²¹²Kövecses 239.

²¹³Lakoff and Núñez's discussion of the FICTIVE MOTION metaphor provides a potential connexion to Rot-

insights into CMT at a more general level, the coherences between Lakoff and Rotman's theories of mathematics suggest that even this relatively restricted partnership could be very fruitful.

What conclusions can be drawn from the above discussion? First, there is a good reason that many contemporary theorists — including this author — fundamentally view mathematics as practice: whereas there is widespread disagreement about mathematical ontology, nearly everyone can agree that humans engage in an activity called mathematics and, moreover, this activity is generally successful and beneficial. Further motivation for the mathematics-as-practice perspective comes from its ability to bypass certain puzzles and pitfalls that make the traditional theories untenable. While there are a variety of ways to understand mathematical practices, a naturalistic approach that understands mathematical activity as one among many human activities is appealing, as it roots itself in the vast wealth of contemporary scientific results. The trick then becomes explaining the distinctive character that seems to make mathematics unique among human practices: its universality, precision, consistency, and so forth. One of the chief puzzles a naturalistic, practice-oriented approach must address is what is going on when mathematicians seem to refer to abstract, empirically implausible entities (infinities, for example). A promising approach is to explain these aspects of mathematics in terms of metaphor; in support of this idea, most people would concede that metaphors are clearly invoked in mathematical communication/instruction, such as when Hilbert's hotel is used to discuss the properties of infinity.²¹⁴ Much of this document has aimed to provide support for the idea that metaphor plays an important role in mathematics.

Discussion of Lakoff and Núñez's embodied mathematics forms the core of this chapter because it is a naturalistic, practice-oriented theory that makes heavy use of conceptual metaphor in explaining how even the most abstract and arcane results of mathematics are

man's Agent that could help get this work underway (Lakoff and Núñez, "The Metaphorical Structure of Mathematics," 53–5).

²¹⁴David Hilbert used the idea of a hotel with infinitely many rooms in his lectures to explore the seemingly paradoxical nature of infinity by showing that even if there is no vacancy and every room is full, the hotel can still accommodate new guests — even an infinite number! (Hilbert, *David Hilbert's Lectures* 730). Even if people concede that metaphor is frequently invoked in mathematical discussion, this does not necessitate that they accept it as a constitutive and ineliminable.

built up from basic human experiences and biological capacities. While the Lakovian perspective provides a promising starting point given the observations of the previous paragraph, the theory is admittedly in its preliminary stages and is not without flaws; this chapter should not be read as a wholehearted endorsement of Lakoff. There is definitely room for improvement and development in the Lakovian approach; accordingly, several other promising contemporary approaches to mathematics were also considered. However, the Lakovian approach has one additional compelling property: CMT is not only a theory of mathematics, but is a general theory of concepts and, therefore, is self-applicable; that is, Lakoff gives us not only a theory of mathematics but also a theory of theories of mathematics. While the traditional theories of mathematics were all rejected as flawed, each was also seen to capture some important aspect of mathematical practice. Metaphor is a device that allows one to bring useful correspondences to the fore while obfuscating problematic and disanalogous aspects and, using conceptual metaphor, Lakoff and Núñez incorporate the fundamental insights of the traditional theories into embodied mathematics. Thus, while reconciliation with rival theories was seen as impossible from within any of the traditional perspectives, Lakoff provides a mildly relativist framework within which even radically contradictory understandings can legitimately coexist to some extent. Lakoff and Núñez explicitly incorporate important aspects of platonism and formalism into embodied mathematics as conceptual metaphors; this dissertation continues this trend by establishing coherences between embodied mathematics and some rival contemporary theories, and by speculating about the core metaphors that may constitutively underlie these rivals. Lakatosian quasi-empiricism, Yablonian figuralism, and Rotmanian semiotics all have potential to enrich embodied mathematics and help it overcome some of its key deficiencies, including accounting for the historical and symbolic/linguistic aspects of mathematics, whether they are found to be outright consistent with the Lakovian approach or only metaphorically coherent. Much work remains if the suggested patchwork theory is to become a reality.

It must be remembered that the theorizing in this chapter is primarily descriptive rather than prescriptive. It recognizes the amazing success of most mathematical practices, and does not seek to alter them by, say, forbidding reference to abstract entities or reducing the level of formal rigor. The prescriptions it does make are primarily aimed at philosophers

of mathematics, suggesting that the paradigm shift to a practice-oriented approach is legitimate and useful, for example. There is, however, one aspect of mathematical practice that should be changed: the distaste and crippling confusion experienced by a significant proportion of people when they engage with mathematics. While the highly formalized mathematics performed by professional mathematicians is very effective in communicating to the mathematically adept, it will generally be ineffective in communicating to the layperson. A prototypical version of such communication occurs between mathematics teachers and their pupils. The recognition that conceptual metaphors underlie mathematical reasoning provides potential inspiration for alternative modes of communication and instruction that are more effective and avoid contributing to the widespread dislike of mathematics. It is perhaps unsurprising that schoolteachers are perhaps the most vociferous supporters of *Where Mathematics Comes From*, and that novel approaches to teaching mathematics based in conceptual metaphor are now occasionally implemented.²¹⁵ It would be nice to see trials of a conceptual-metaphor-based approach in the post-secondary mathematics classroom, where both professors and students are often dissatisfied with the educational process.

The main goal of this dissertation is to investigate the relationship between mathematics and metaphor. Thus far, the focus has been on the metaphor side of the equation, considering what role, if any, metaphor plays in mathematics. Answering this question even somewhat satisfactorily required a lengthy discussion about the nature of metaphor, ultimately concluding that contemporary understandings of metaphor as fundamentally conceptual rather than merely linguistic are superior. Proceeding from that conclusion, the present chapter has argued that conceptual metaphor helps explain not only mathematical practice but also the philosophy of mathematics, providing a mechanism for amalgamating the positive aspects of various theories of mathematics while avoiding contradiction. The final core chapter of this dissertation goes against the flow of the previous three chapters and investigates the extent to which metaphor can be understood in terms of mathematics.

²¹⁵See, for example, Danesi (2007).

CHAPTER 5

METAPHOR IS MATHEMATICAL

Every discourse on metaphor originates in a radical choice. . . If it is metaphor that founds language, it is impossible to speak of metaphor unless metaphorically. Every definition of metaphor, then, cannot but be circular. The other option is that metaphor is a deviation from normal, preestablished rule-governed language.¹

— Umberto Eco

The modeling capacity of mathematics is astounding. Humans constantly understand the world in mathematical terms, from counting the bottles of beer left in the fridge to describing the properties of subatomic particles. While some experiences are perhaps not well-suited to mathematical explanation, there is very little that is *entirely* beyond the purview of mathematics.² Almost all empirical science, for example, involves some quantitative aspect. It is clear at any rate that language and communication are prime candidates for mathematical modeling. An important question thus arises: to what extent can metaphor be understood in terms of mathematics?

There are three possible answers to this question: not at all, somewhat, and entirely. The first answer is not only uninteresting and would make for a very short chapter, but also seems demonstrably false given the number of researchers making progress in this area. The third answer is what many of the aforementioned researchers are aiming for, but attaining a

¹Eco, *Semiotics and the Philosophy of Language* 88.

²As John Barrow puts it, foreshadowing some of my upcoming sentiments about applied math:

Mathematics is also seen by many as an analogy. But it is implicitly assumed to be the analogy that never breaks down. Our experience of the world has failed to reveal any physical phenomenon that cannot be described mathematically. That is not to say that there are not things for which such a description is wholly inappropriate or pointless. Rather, there has yet to be found any system in Nature so unusual that it cannot be fitted into one of the strait-jackets that mathematics provides (Barrow 21).

robust mathematical model for generating and interpreting metaphors seems a long shot given the complexity and elusiveness of the subject matter. Eco agrees with and surpasses this sentiment: “No algorithm exists for the metaphor, nor can a metaphor be produced by means of a computer’s precise instructions, no matter what the volume of organized information to be fed in.”³ Whatever one’s opinion on whether this ambitious goal can possibly be achieved, it is certainly beyond the expertise of this author and the scope of this document.

The primary goal of this chapter is to provide a partial understanding of metaphor using mathematical metaphors. While authors from Nicomachus to Lakoff and Johnson have claimed that mathematical modeling and application are metaphorical in nature, the metaphors invoked in this chapter will be of a less rigorous variety.⁴ That is, while this chapter will invoke a variety of mathematical concepts — some which may be unfamiliar to those with no advanced mathematics training — it should *not* be viewed as a work of mathematics. The imprecise use of precise concepts and theories is distasteful to many specialists, but does not seem inherently problematic if the imprecision is acknowledged. Though no mathematical models for metaphor will be offered in this chapter, it is possible that the metaphors introduced here could one day be developed into something more rigorous and exact. Any such developments would be welcome, but the understanding of metaphor presented in this chapter is meant to be useful in its own right, not merely as a launching pad for precise mathematization.

5.1 Computational Linguistics

For the sake of completeness, it is worth briefly considering the history of attempts to understand language and metaphor mathematically; readers who find the metaphorical approach distasteful may find something more palatable in this section. The connexions between mathematics and language extend far back into human history: because language provides a rich source of patterns, and because mathematics is aptly described as “the science of patterns,” it seems inevitable that mathematics would be applied to language structure.⁵ Language pat-

³Umberto Eco, “Metaphor, Dictionary, and Encyclopedia,” *New Literary History* 15.2 (1984): 269.

⁴Athenaeus 290; Lakoff and Johnson, *Philosophy in the Flesh* 91.

⁵Keith Devlin, *Mathematics: The Science of Patterns* (New York: Scientific American Library, 1997), 65.

terns are perhaps most blatantly apparent in poems, linguistic works observing structuring constraints on metre, rhyme scheme, verse shape, and so forth; the creation of many forms of poetry involves numerical considerations, particularly in the form of syllable counting in styles as diverse as haiku and iambic pentameter. Another connexion between language and mathematics is that, prior to the introduction of Hindu-Arabic numerals to Europe around 1000 CE, glyphs from the Hebrew, Greek, and Latin alphabets did double duty, serving both as letters and as numerals.⁶ Most readers will be somewhat familiar with this phenomenon from studying Roman numerals in grade school (I=1, V=5, X=10, L=50, and so on), even if this connexion was not explicitly recognized previously. This association between numbers and letters was exploited in numerological practices which sought to understand words by way of the numerical result of summing the values associated with their constituent letters; this was called *gematria* in the Judiac tradition and *isopsephy* in the Greek.⁷ Today, numbers are still associated with letter symbols, though for less spuriously arcane purposes. ASCII, Unicode, and other encoding schemes associate alphabetic symbols with numerical codes that allow computers to handle text.⁸ While every personal computer is capable of decoding ASCII, other numerical ciphers are employed when sensitive messages must be conveyed securely. While these symbol-oriented connexions between numbers and language are interesting, and set the stage for higher-level modeling, they do not help with understanding metaphorical phenomena.

There have been many attempts to mathematize higher-level syntactic and semantic aspects of language. Early attempts typically aimed to reduce confusion and conflict, appealing to mathematics to introduce precision and clarity to language; these efforts were generally prescriptive rather than descriptive and, if successful, probably would have involved the elimination of metaphor from most discourse. Aristotle's development of syllogistic logic in the *Organon* is arguably the first serious attempt to present a systematic theory of reasoning in

⁶Barrow 93.

⁷Ivor Grattan-Guinness, *The Norton History of the Mathematical Sciences* (New York: W.W. Norton and Company, 1997), 125. The "Revelation" that the Number of the Beast is 666 is perhaps the most widely familiar example of gematria.

⁸TEX, the typesetting system used to produce this document, works somewhat differently from these encoding schemes, but still ultimately must use numbers to encode features of text.

order to avoid error and sophistry.⁹ In the *Discourse on Method*, Descartes holds the evidentness of mathematical proofs as the ideal that all other reasoning should aspire to: “Those long chains of utterly simple and easy reasonings that geometers commonly use to arrive at their most difficult demonstrations had given me occasion to imagine that all the things that can fall within human knowledge follow from one another in the same way.”¹⁰ This line of thinking culminates in Leibniz’s project to develop the *characteristica universalis*, a symbolic universal language combining the scope of natural language with the precision and calculability of arithmetic and algebra:

If we had some exact *language* ... or at least a kind of *truly philosophical writing*, in which the ideas were reduced to a kind of *alphabet of human thought*, then all that follows rationally from what is given could be found by a *kind of calculus*, just as arithmetical or geometrical problems are solved ... I can demonstrate with geometric rigor that such a language is possible, indeed that its foundation can be easily laid within a few years by a number of cooperating scholars.¹¹

In such a language, disagreements would be resolved not by mediation or compromise, but by calculation: “when a controversy arises, disputation will no more be needed between two philosophers than between two computers. It will suffice that, pen in hand, they sit down to their abacus and ... say to each other: *let us calculate*.”¹² Central to this project is a conviction that all human ideas are made up from a set of primitive concepts, just as every integer has a unique prime factorization.¹³ By directly symbolizing these primitives as naturally as possible (drawing on inspiration from Chinese characters and Egyptian hieroglyphics) and providing precise mechanisms of composition and derivation, Leibniz hoped to create an external isomorphic copy of our conceptual system that would eliminate all errors of reasoning apart from the occasional simple calculation mistake.¹⁴ In places, Leibniz proposes using

⁹Robin Smith, “Aristotle’s Logic,” *Stanford Encyclopedia of Philosophy*, 28 Jan. 2014.

¹⁰René Descartes, *Discourse on Method*, Trans. Donald Cress, 3rd ed. (Indianapolis: Hackett, 1998), 54.

¹¹Gottfried Wilhelm von Leibniz, “On the Universal Science: Characteristic,” *Monadology and Other Philosophical Essays*, Ed. and trans. Paul and Anne Martin Schrecker (New York: Bobbs-Merrill, 1965), 12; emphasis his.

¹²Leibniz 14; emphasis his. Of course, the use of the word “computers” in this translation refers to the profession and not the electronic devices ubiquitous in present day, though making such an anachronistic mistake seems to make little difference in this particular case. Leibniz’s stepped reckoner was the first mechanical calculator (theoretically) able to perform all four arithmetic operations, and thus was an important predecessor of the electronic computer (Barrow 128).

¹³There could be interesting connexions between Leibniz’s primitives and Lakoff’s basic-level concepts that are worth exploring.

¹⁴Leibniz 18.

prime numbers rather than representationally faithful logograms to represent the primitive concepts, and representing compound concepts with characteristic numbers formed by taking the product of their primitives — an approach very similar to that implemented by Gödel in proving his incompleteness theorems.¹⁵ Elsewhere, Leibniz suggests that the *characteristica universalis* would have to be able to accommodate probabilistic considerations, anticipating to some small extent contemporary statistical linguistic models.¹⁶ Though Leibniz apparently abandoned this project, his suggestions were ahead of their time and importantly influenced later attempts to mathematize language.

The nineteenth century saw several important developments that led to a change in the goals of language mathematization projects, including significant work in axiomatic systems, formal symbolic logic, mechanical computation and communication devices, and linguistics. Emphasis began to split between trying to develop new mathematically precise languages oriented from concept-to-sign and attempting to mathematically describe extant natural language use from sign-to-concept; rather than trying to eliminate error by artificially constraining language, researchers attempted to include the double-edged complexities of natural language that generate both errors and richness into their models. These efforts to mathematically describe the mechanisms of language importantly have application in machine-translation and artificial-intelligence research; the explosive technological development of the twentieth century correspondingly induced a significant increase in linguistic modeling. Models of language have become increasingly technical and sophisticated, and yet many of them belong to the tradition that marginalizes metaphor as derivative and deviant and attempts to eliminate it through paraphrase. Some of the most promising contemporary approaches — such as the one underlying Google Translate — break with tradition and use statistical rather than algorithmic models, relying on unfathomably massive collections of data and blistering computing speed to mechanically generate sentences.¹⁷ Such models may be able

¹⁵Barrow 128. Interestingly, even though his proof that some propositions are undecidable seems to constitute a major problem for Leibniz’s project, Gödel maintained that the *characteristica universalis* was viable and, furthermore, that a conspiracy systematically suppressed publication of the portion of Leibniz’s oeuvre dedicated to developing the universal language (Dawson 166). Given that much of Leibniz’s writing remains unpublished, it is conceivable that important results yet lurk undiscovered in the archives.

¹⁶Leibniz 15.

¹⁷“Inside Google Translate,” *Google Translate*, 28 Jan. 2014.

to translate and generate linguistic metaphors, yet do not seem to contribute much to *understanding* metaphor.¹⁸ However, this should not be understood to imply that no attempts to model metaphor have been made.

Several researchers have specifically attempted to mathematically model metaphorical reasoning; such authors tend to subscribe to some non-traditional view of metaphor-as-conceptual. As discussed above, the details of these theories are beyond the scope of this chapter and the expertise of this author: I am no computational linguist or informaticist. A brief list of a few noteworthy efforts follows for those readers interested in pursuing a more-precise mathematical approach to metaphor than is presented below. Perhaps the best known and most widely implemented model is Dedre Gentner’s Structure Mapping Engine (SME), an algorithmic model of analogy that emphasizes higher-order structural similarities over shallow similarities; Gentner conceives of metaphor as invoking analogical reasoning.¹⁹ A variety of competing predictive models are offered by Keith Holyoak and his coauthors: ACME (Analogical Constraint Mapping Engine), ARCS (Analogical Retrieval by Constraint Satisfaction), CWSG (Copy With Substitution and Generation), LISA (Learning and Inference with Schemas and Analogies), and ECHO (Explanatory Coherence by Harmony Optimization).²⁰ Interestingly, the more recent effort BART (Bayesian Analogy with Relational Transformation) integrates a probabilistic component.²¹ Douglas Hofstadter and his Fluid Analogies Research Group criticize Gentner, Holyoak, and their various contemporaries for bypassing representation building in their models: “if appropriate representations come presupplied, the hard part of the analogy-making task has already been accomplished.”²² The models generated by the Fluid Analogies Research Group — Seek-Whence, Jumbo, Numbo, Copycat, Tabletop, and Letter Spirit — accordingly focus on avoiding this criti-

¹⁸This is not entirely fair. Even with my extremely limited understanding, it seems clear that such models must include a mechanism for sifting through the data for similar or *analogous* cases; studying the mechanisms employed for this purpose could be extremely enlightening.

¹⁹See Gentner (1983), Falkenhainer, Forbus, and Gentner (1989), and Gentner et al. (2001) to start; there are many, many publications on SME if one is compelled to investigate this theory at length.

²⁰Keith J. Holyoak, “Overview of Research Career,” *Keith Holyoak’s Web Page*, UCLA Reasoning Lab, 18 Aug. 2013. For more information, see his extensive publication list on his website; the original paper is Holyoak and Thagard (1989).

²¹See Lu, Chen, and Holyoak (2012).

²²David Chalmers, Robert French, and Douglas Hofstadter, “High-level Perception, Representation, and Analogy,” *Fluid Concepts and Creative Analogies* (New York: Basic Books, 1995), 182.

cism.²³ Though there are many additional metaphor modeling projects currently under way, one other stands out as particularly relevant here: a team of informaticists from the University of Edinburgh is working on a computational model based in Information Flow theory with the specific aim of modeling Lakoff and Núñez’s arithmetic grounding metaphors.²⁴ While such modeling projects are interesting and admirable, their details are outside the purview of this chapter.

There are several reasons that this chapter provides only a cursory list of these modeling projects. First, as mentioned above, much of the work being done is outside of my current scope of expertise: I am not in a position to summarize, let alone evaluate, their efforts. Second, several noteworthy scholars have concerns that all such modeling projects are inherently flawed. Lakoff’s complaint seems to stem from the fact that functions, by definition, can have at most one output per input: “the contemporary theory of metaphor is at odds with certain traditions in symbolic artificial intelligence and information processing psychology... those traditions must characterize metaphorical mappings as an algorithmic process, which typically takes literal meanings as inputs and gives a metaphorical reading as an output. This runs counter to cases where there are multiple, overlapping metaphors in a single sentence, and which require the simultaneous activation of a number of metaphorical mappings.”²⁵ The details of Eco’s objection are a bit obtuse, being couched in technical semiotic jargon, but his conclusion is crystal clear (and thus worth repeating): “No algorithm exists for the metaphor, nor can a metaphor be produced by means of a computer’s precise instructions, no matter

²³See Hofstadter and the Fluid Analogies Research Group (1995). One regret I have in writing this dissertation is that, for various reasons, I have not been able to incorporate more of Hofstadter’s work. Much of his thought on metaphor and analogy seems coherent (and even consistent!) with the Lakovian perspective and — more importantly — with my own. He unabashedly claims that “every concept we have is essentially nothing but a tightly packaged bundle of analogies” (Hofstadter, “Epilogue” 500), and, correspondingly, that “*analogy-making lies at the heart of intelligence*” (Hofstadter, *Fluid Concepts and Creative Analogies* 63). He, like me, started as a graduate student in mathematics but ended up changing disciplines when he discovered that “I had always thought that I was a pretty abstract thinker, but... in fact, all of my thoughts are very concrete. They are all based on images, analogies, and metaphors. I really think only in concrete ideas, and I found that I couldn’t attach any concrete ideas to some of the mathematics I was learning.” (Hofstadter, “Analogies and Metaphors to Explain Gödel’s Theorem” 98). Had things gone only slightly differently during the conception of my project, Hofstadter could well have formed the core of this dissertation instead of Lakoff; I will certainly be integrating more of his work into my future projects.

²⁴See Guhe, Smail, and Pease (2009).

²⁵Lakoff, “The Contemporary Theory of Metaphor” 249.

what the volume of organized information to be fed in.”²⁶ Whether these concerns are well-founded is of minor relevance here. Even if a thorough and robust computational model of metaphorical reasoning and language were obtained, it would not thereby constitute the *only* legitimate approach to understanding metaphor! While a precise algorithmic understanding would be ideal for programming computers to generate and/or interpret metaphors, it seems unlikely that a thorough computational model would be concise or transparent, making it less beneficial for aiding the understanding of most human individuals. A good metaphor, on the other hand, can efficiently and effectively capture the essence of a complex idea and simplify communication. For example, while “Crotone is located at 39° 05′ N 17° 07′ E” is precise, it is also theoretically-dense and therefore inaccessible to someone unversed in the language of the global coordinate system, whereas Walton’s “Crotone is on the arch of the Italian boot” conveys immediate understanding to anyone who has the most basic knowledge about the globe and footwear.²⁷ Although both kinds of understanding are valuable, it is the imprecise but succinct metaphorical variety that this chapter aims to generate.

5.2 Mathematical Metametaphors

In chapter 2, it was shown that from its very origins, METAPHOR has itself been understood by way of metaphors — what I have been calling metametaphors. The remainder of this chapter is dedicated to developing metametaphors with mathematical source domains. A brief defense and justification of this approach is warranted. At the beginning of the chapter, it was noted that some mathematically inclined specialists may find the imprecise application of mathematical concepts distasteful and problematic; however, the explicit acknowledgment that this non-standard use of mathematical concepts is not intended to be understood as a work of mathematics seems to vindicate its relative lack of rigor and precision. In particular, this approach should not be taken as endorsing — or even condoning — sloppy, inconsistent mathematics. Consider the following analogous case: intentionally and deliberately using metre sticks to measure *weight* is an atypical practice that employs a tool for a purpose other

²⁶Eco, “Metaphor, Dictionary, and Encyclopedia” 269.

²⁷Walton, “Metaphor and Prop Oriented Make-Believe” 40.

than the one it was devised for.²⁸ However, such an act of weighing is not invalidated simply by virtue of its atypicality, and certainly does not constitute an unskilful or “sloppy” act of measuring *length*! A related concern is that employing metaphor to understand metaphor leads to vicious circularity. However, the kind of circularity involved does not seem to be necessarily problematic. For example, a micrometer-manufacturing machine may have its dimensions and tolerances measured by one of the micrometers it has itself created. Insofar as all human understanding involves the brain, our understanding of neurons is neuronal.²⁹ Gödel numbers encode arithmetic expressions as integers, thereby, in a sense, using arithmetic to understand arithmetic. So it is for metaphor:

Every scientific theory is constructed by scientists — human beings who necessarily use the tools of the human mind. One of those tools is conceptual metaphor. When the scientific subject matter is metaphor itself, it should be no surprise that such an enterprise has to make use of metaphor, as it is embodied in the mind, to construct a scientific understanding of what metaphor is.³⁰

Thus, there seems to be a sufficient gap between metaphor qua capacity and metaphor qua empirical phenomenon to avoid viciousness, though extra vigilance in watching out for question-begging is warranted. Finally, I concede that the metametaphors developed below may be inaccessible to a large proportion of the population, because the mathematical ideas invoked may lie beyond the competency of the average person, and because those who do have an understanding of the mathematical ideas involved are more likely to have an aversion to their being used in this way.³¹ Worries that a given approach may not be widely accessible or popular should not be considered an adequate reason to abandon inquiry; while I hope that at least some readers find the ideas presented in this chapter useful or at least interesting, that I find them useful and interesting provides sufficient grounds to proceed.

The use of mathematical metametaphors is not unprecedented. Lakoff and Johnson say that their original conception of conceptual metaphor relied on the metaphor METAPHORS ARE MATHEMATICAL MAPPINGS:

²⁸For example, at the time of writing this, I weigh 653 metersticks.

²⁹Though they are making a different point, this example was inspired by Lakoff and Johnson (*Philosophy in the Flesh* 103).

³⁰Lakoff and Johnson, *Metaphors We Live By* 252.

³¹Of course, people committed to a traditional conception of metaphor as an eliminable linguistic garnish are also unlikely to find this work particularly compelling.

Our first metaphor for conceptual metaphor came from mathematics. We first saw conceptual metaphors as mappings in the mathematical sense, that is, as mappings across conceptual domains. This metaphor proved useful in several respects. It was precise. It specified exact, systematic correspondences. It allowed for the use of source domain inference patterns to reason about the target domain. Finally, it allowed for partial mappings. In short, it was a good first approximation.³²

However, they eventually saw METAPHORS ARE MATHEMATICAL MAPPINGS as problematically dis-analogous, failing to capture a crucial feature of metaphor: namely, that “[m]athematical mappings do not create target entities, while conceptual metaphors often do.”³³ Lakoff and Johnson went on to explore other metametaphors, including as METAPHOR IS SLIDE PROJECTION, before adopting their current neural approach to metaphor.³⁴ However, their abandonment of METAPHORS ARE MATHEMATICAL MAPPINGS seems premature. Mathematics does have some resources for dealing with the creation of new entities which may circumvent Lakoff and Johnson’s concerns; moreover, even if mathematics did not possess any such resources, this would seem to indicate a promising direction for future mathematical development rather than a dead end. The metaphor METAPHORS ARE MATHEMATICAL MAPPINGS has more to offer than Lakoff and Johnson realized; a more detailed exploration of some mathematical examples and their metaphorical entailments will show that this metametaphor still has the potential to make an important contribution to how conceptual metaphor is conceived.³⁵

The plan for the remainder of this chapter is to consider some mathematical analogues of the core salient properties of metaphor that were discussed earlier in the dissertation, thereby hopefully rejuvenating and improving upon the Lakovian mathematical metametaphor in the process. One point that everyone from Aristotle to Lakoff agree on is that metaphor involves a comparative or interactive relationship between two things or ideas; this most basic observation is at the heart of the METAPHORS ARE MATHEMATICAL MAPPINGS metaphor. However, mathematical mappings come in a variety of flavours, some of which are a better fit for

³²*Metaphors We Live By* 252.

³³Lakoff and Johnson, *Metaphors We Live By* 252.

³⁴Lakoff and Johnson are not the only ones to invoke mathematical metametaphors: recall Yablo’s logarithm analogy for understanding games of make-believe, including metaphor (“Go Figure” 182).

³⁵It should be noted that Lakoff tends to focus his attention on entrenched “metaphors we live by” that are constitutively integrated into our conceptualizations. The metaphors offered below are creative and novel, not ubiquitous and conventionalized. That is, the following metaphors have been *invented* to help make sense of the difficult notion of METAPHOR, not *discovered* as an existing part of the common understanding.

metaphor than others. First, because we understand metaphors as inference-preserving, it seems metaphors are more analogous to morphisms — structure-preserving mathematical mappings — than to simple binary relations; the prototypical morphism is the function, but the class also includes homomorphisms and functors.³⁶ While mere relations are too unstructured to be a good fit for metaphor, other mappings are too structured. Metaphors necessarily cross domains and involve disanalogy, and are therefore not identity mappings; more generally, the disanalogy requirement suggests that only a proper subset of the source domain is mapped by the metaphor. There is neither a requirement that a metaphor subsume every part of the target domain, nor a prohibition against distinct components of the source domain mapping to the same component of the target domain; metaphors are therefore typically neither surjective nor injective. The asymmetry noted in many metaphors manifests mathematically as non-invertibility of the mapping. Even the functionhood of some metaphor maps might be questioned, as the same metaphor could be interpreted differently by different people (or by the same person at different times), meaning a single element of the source domain might map to multiple targets; to keep this discussion fruitful and accessible, metaphorical mappings will be assumed to be functions.³⁷ Thus far, more has been said about what metaphors are not than what they are; in general, their inference-preserving nature means they are best understood as functions or, perhaps, homomorphisms. To delve further, we must consider what metaphors map between.

Conceptual metaphors connect two concepts, or conceptual domains. Mathematical maps connect two mathematical objects or structures. The question is, what mathematical structures are usefully analogous to concepts? Discussion in chapter 3 showed that humans employ a broad and diverse range of concepts, and therefore providing a theory that adequately accounts for them all is a staggeringly difficult task. It thus seems unlikely that any metaphor

³⁶The main difference between these morphisms is what kind of mathematical objects they map between. For example, group homomorphisms are operation-preserving mappings between groups, real functions are structure-preserving mappings on the set of real numbers, and functors are morphism-composition-preserving mappings between the abstract structures known as categories.

³⁷One might try to preserve functionhood by considering different interpretations as different mappings, or by exploiting the lack of simultaneity, or by attaching probabilities to the various targets. These are promising approaches, but it nonetheless seems possible that a single interpretation could simultaneously map a single source to multiple targets, though no example springs to mind. There is a theory of multifunctions that could be applied here if warranted.

provided here will be able to do more than capture some generalities about concepts. First, however, consider that CONCEPTS ARE INTEGERS does not seem to be an especially apt metaphor despite a certain amount of historical popularity. As discussed above, Leibniz's *characteristica universalis* project aimed to find “conceptual primes” that could be combined into more-complex concepts via multiplication. Conceptual atomists adopt a similar stance — recall from chapter 3 that they hold that, just as unfactorable prime numbers can be multiplied to make composite numbers, concepts are structureless primitives that can be composed into complexes — but without Leibniz's emphasis on computation. However, the posit of primitive concepts with no internal structure is incompatible with the understanding of concepts found in CMT, where even the most basic, image-schematic concepts have innards.³⁸ The idea that concepts themselves are structured is certainly not unique to CMT; for example, Douglas Hofstadter makes the claim that “words and concepts are far from being regularly shaped convex regions in mental space; polysemy (the possession of multiple meanings) and metaphor make the regions complex and idiosyncratic.”³⁹ It thus seems that something more mathematically complicated than numbers is needed here: at the very least, insofar as we understand conceptual metaphors as structure-preserving mappings, the objects they map between should possess some structure to preserve.

Though they are the first (and possibly only) kind of mathematical objects to come to mind for most people, numbers are far from the only mathematical entities available for consideration. In *Metaphors We Live By*, Lakoff and Johnson claim that many concepts possess a unified multidimensional structure that we impose on our experiences in order to classify and understand them.⁴⁰ To the extent that this is deemed plausible, it makes sense to understand a concept as a structure existing inside some multidimensional space. Why not understand the concept as the entire multidimensional space? There are at least two reasons. One, concepts are generally not static totalities but tend to change, grow, and develop over time, potentially even expanding into new dimensions; in particular, if metaphors are to be able to create target entities as Lakoff insists, they must have some uncharted room to do so in. Positing that concepts are contained in infinite spaces ensures that this potential for

³⁸Lakoff, *Women, Fire, and Dangerous Things* 279.

³⁹“Epilogue” 511.

⁴⁰*Metaphors We Live By* 80–3.

development is never exhausted. Two, in some sense a proper subset of an infinite space can actually possess more interesting structure than the space itself insofar as the former has additional constraints. This observation also coheres with Hofstadter’s claim about concepts possessing complicated shapes. While an entire infinite space such as \mathbb{R}^n is arguably too big, complete, and symmetrical an entity to represent a concept, singleton points or vectors within such a space are also likely inapt insofar as they are nearly as limited in structure as numbers. One more general observation about conceptual space is salient here. Even though it seems clear that concepts can possess vastly different amounts of complexity and dimensions of structure, and even though concepts like COFFEE, TELEPORTATION, and FEELINGS seem to be distinctly distinct, there are good reasons to think of all of a person’s concepts as existing within a single conceptual space. First, this understanding avoids multiplying entities unnecessarily: it is possible to imagine a single space that is vast and dimensioned enough to accommodate them all. Second, it seems rather natural to consider a person’s concepts to exist within a single space insofar as they exist within a single mind. And third, understanding concepts as occupying a single space is coherent with our conceptualization of concepts as being more or less distant from each other: GOOSE is usually thought of as *closer* to DUCK than it is to AARDVARK or ADVERB.

This observation suggests that conceptual space may be a *metric space*; that is, that a “distance” function is defined on it. Such structure could be used to help explain the typicality effects observed by Rosch: PENGUIN could be understood as more distant from BIRD than ROBIN is, for example. However, there are also reasons to think that the notion of conceptual distance does not possess all of the necessary definitional characteristics of a metric. As a side effect of the typicality gradient, one might see PENGUIN as closer to ROBIN than ROBIN is to PENGUIN, for example, because, as it were, the first distance assessment takes the measurer deeper into the concept BIRD while the second measurement leads one out to its periphery. Thus, unlike a metric, conceptual distance seems to lack symmetry with respect to its arguments. For similar reasons, it is not clear that the triangle inequality holds; depending on how addition of “distances” is interpreted, ROBIN and PENGUIN might plausibly be seen as closer together if one goes by way of BIRD instead of directly.⁴¹ While it seems uncontroversial

⁴¹The triangle inequality: if d is a metric on a space X then $d(x, y) \leq d(x, z) + d(z, y) \forall x, y, z \in X$.

that there is no distance between a concept and itself, there may also exist cases where two distinct concepts are judged to have zero distance between them, at least within a context where the differences are irrelevant to the purpose at hand: the distance between DOMESTIC GOOSE and WILD GOOSE might be zero at lunchtime, though this is a controversial claim. Of course none of this is definitive; despite these observations, it may be possible through careful mathematical work to construct a well-defined conceptual metric. For example, there may be multiple notions of distance at work here that could be teased apart. Even if the idea of conceptual distance cannot be made well-behaved enough to be considered a full-fledged metric, this does not mean it is not useful or even that it is mathematically illegitimate — it might simply be a restricted kind of distance that fails to satisfy one or more of the conditions of metrichood, or it could induce a topology on conceptual space, or it could do none of these things and still be an interesting property of the space. In any case, conceptual distance is one kind of structure that could be preserved by metaphorical mapping. Enough about conceptual space, on to concepts proper.

So far, concepts have been described as internally structured, complicatedly shaped entities coexisting within a multidimensional infinite space endowed with some kind of “distance” function. Additionally, it was noted that concepts are not static entities but rather are dynamic, changing and developing over the course of a person’s life. Each successive version of a particular concept depends on a variety of factors, including experiential inputs, other conceptual developments, and the previous version of the concept.⁴² This observation suggests a very promising metaphor: CONCEPTS ARE FUNCTIONS. If concepts are functions, then conceptual space is a function space, a vector space whose elements are functions.⁴³ Concepts can thus be seen as both individual points of a space as well as entities possessing shape and internal structure in their own right, certainly far more structure than an individual n-tuple;

⁴²It is worth noting that the word “version” has connotations of discreteness, but it is probably best to understand concepts as being continuously dynamic.

⁴³Actually, given the idea that concepts develop and change over a person’s lifetime, we should probably think of concepts as time-dependent paths through a function space. Moreover, these trajectories should probably be treated as dynamical systems insofar as each successive version of the concept significantly depends upon the previous one. However, for the rest of this chapter, we will primarily think of concepts as single elements in a function space for two reasons. First, as mentioned in chapter 4, CMT has a synchronic bias. Second, this will simplify the mathematics considerably, hopefully allowing this chapter to remain at least somewhat accessible.

indeed, functions are interpreted set theoretically as sets of ordered pairs (that is, 2-tuples) of the form (input, output). Drawing inspiration from Hofstadter's idea of conceptual *shape*, recall that most of the functions a person deals with in their early mathematical training can be represented by plotting the set of ordered pairs to form a *graph* of the function; for example, the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ can be represented in the Cartesian plane as a parabolic curve opening upward whose vertex is located at the origin. In principle, the idea of graphing can be extended to functions defined on more complicated domains and codomains, though creating adequate visual representations of such graphs may be impossible due to the constraints of the Euclidean 3-space we inhabit.⁴⁴ Having understood concepts as functions — and, in particular, as the graphs of those functions — it makes sense to next ask what the domain and codomain of concepts are. Here, I take a metaphorical liberty and assume them to both be \mathbb{R} , even though this conflicts with the understanding of concepts as multidimensional (Lakoff and Johnson's analysis of *WAR* requires at least *six* dimensions, for example).⁴⁵ There are good reasons for doing this. Concepts are only being addressed here because they form a necessary component of the understanding of metaphor under consideration. Given that *CONCEPT* is a massively complicated and difficult concept to understand, it would be easy to get caught up in trying to create an increasingly detailed metaphorical understanding of this peripheral notion, thereby losing sight of the primary objective of this discussion. Imposing the real numbers as the domain and codomain of concepts (and intentionally leaving these axes uninterpreted) allows focus to return to metaphor. While choosing a domain with more dimensions would be more faithful to Lakoff (and to my own intuitions), the simpler choice made here is communicatively preferable as it makes the necessary mathematics more accessible and allows the reader to visualize functions in a familiar setting. In principle, interested parties should be able to generalize the discussion below to a variety of different domain-codomain pairs that would be more apt.

⁴⁴Let $f : X \rightarrow Y$ be a function. The *domain* of f is X , the set of inputs for which f is defined. The *codomain* of f is Y . The *image* or *range* of f is $f(X)$, the set of all outputs of f . The image of a function is not necessarily identical to its codomain, though $f(X) \subset Y$.

⁴⁵*Metaphors We Live By* 80–1.

If concepts are functions, then metaphors map functions to functions in a structure-preserving way.⁴⁶ To help gain some insight into what the nature of this mapping could be, recall that metaphor is frequently conceived as an act of *seeing-as*: seeing Italy as a boot, seeing Achilles as a lion, “seeing” (experiencing) affection as warmth, and so on. Two situations in mathematics that could be understood as seeing-as spring to mind. First, in linear algebra, points in \mathbb{R}^3 are usually described as linear combinations of the standard basis vectors; for example $(x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$. However, under some circumstances one may wish to change the basis, that is, see the points of the vector space as linear combinations of an alternative basis: $(x, y, z) = x(1, 0, 1) + y(0, 1, 1) + (z - x - y)(0, 0, 1)$. Second, when considering geometric objects, a *symmetry* is a transformation that appears to leave the object unchanged; for example, rotating a square through 90° produces a result indistinguishable from the original situation. Thus, finding symmetries involves transforming an object (by rotating it, reflecting it, shifting it, scaling it, etc.) and considering to what extent the result resembles the original. This kind of seeing-as becomes more interesting as the objects become more complicated and partial alignments of geometric structure become more frequent than total alignments (as happens with aperiodic tilings, for example). That the latter variety of mathematical seeing-as could be relevant here is suggested by Lakoff and Johnson: “Understanding a conversation as being an argument involves being able to superimpose the multidimensional structure of part of the concept WAR upon the corresponding structure CONVERSATION.”⁴⁷ Their METAPHOR IS SLIDE PROJECTION metametaphor developed out of this approach.⁴⁸ Gregory Bateson provides further inspiration to pursue this line of thinking in his discussion of moiré phenomena (when two superimposed patterns generate a third): “it becomes possible to investigate an unfamiliar pattern by combining it with a known second pattern and inspecting the third pattern which they together generate.”⁴⁹ Perhaps metaphor can be seen as involving a superimposition of function graphs?

A substantial mathematical theory exists that spectacularly combines these various ideas. *Fourier analysis* involves seeing functions as sums of sine and cosine waves. That this is math-

⁴⁶Mappings between function spaces are often referred to as *operators*.

⁴⁷*Metaphors We Live By* 81.

⁴⁸Lakoff and Johnson, *Metaphors We Live By* 253–4.

⁴⁹Gregory Bateson, *Mind and Nature* (New York: E.P. Dutton, 1979), 79.

ematically robust depends on the fact that an orthonormal basis of the function space L_2 can be made from sine and cosine waves.⁵⁰ If the function under consideration is periodic, it can be written as a (possibly infinite) Fourier series, that is, as a linear combination of the wave basis vectors. However, a wider variety of square-integrable functions can be handled by way of the *Fourier transform* which produces a continuous rather than discrete interpretation of the function in terms of waves.⁵¹ The Fourier transform has become an extremely valuable tool for scientists and engineers because all empirical waveforms have compact support and are bounded and are therefore square integrable. Real-world applications such as signal storage and compression, noise filtering, and pattern detection involve Fourier analysis. The Fourier transform is a mapping from a function to a function which involves a kind of mathematical seeing-as; perhaps metaphor can be understood as a Fourier transform? There are at least two significant problems with this idea. First, Fourier analysis always involves a seeing-as in terms of sine and cosine functions, whereas metaphor must be able to accommodate a much wider variety of source domains.⁵² Second, the result of applying a Fourier transform to a waveform is not another waveform of the same variety but a frequency spectrum; that is, Fourier transforms are usually understood to map functions defined on a time domain to functions defined on a frequency domain. Applying Fourier analysis to a continuous periodic function produces a discrete set of frequency coefficients; there is thus no meaningful resemblance between the output of a Fourier transform and either its input function or a sine wave. The output of a Fourier transform is not a function graph of the right kind to be a concept, and therefore it cannot be the metaphor mapping.

⁵⁰Rajendra Bhatia, *Fourier Series* (Washington, DC: Mathematical Association of America, 2005), 10. *Orthogonality* of vectors is a generalization of geometric perpendicularity defined in terms of an inner product. A vector is *normal* if it has norm 1; a norm is a function that generalizes the notion of the length or magnitude of a vector, and is also often defined in terms of an inner product. A basis, then, is orthonormal if all its vectors have norm 1 and are mutually orthogonal. L_2 is the function space of square-integrable functions; that is, $L_2 = \{f : \int |f|^2 < \infty\}$. Technically, the orthonormal basis of sines and cosines mentioned exists for a version of L_2 restricted to the interval $[-\pi, \pi]$ (Bhatia 9).

⁵¹The standard Fourier transform is defined by $\hat{f}(t) = \int_{-\infty}^{\infty} f(x)e^{-itx}dx$ (Bhatia 11). Fourier analysis is not confined to \mathbb{R} ; versions of the Fourier transform exist for functions defined on a wide range of spaces and manifolds (Triebel 4).

⁵²The idea that all functions can be understood in terms of waves is analogous to the thought that mathematics provides a universal source domain, that everything in the world can be understood in terms of mathematics. Given this observation, it is thus perhaps not surprising that Jean Baptiste Joseph Fourier himself once said “Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them” (qtd. in du Sautoy).

All is not lost. A few modifications will rectify these difficulties and show that integral transforms provide a useful way to understand metaphor. First, mathematicians have developed and studied a variety of integral transforms akin to the Fourier transform but based upon different analyzing functions. Of particular note are the wavelet transforms, integral transforms similar to the Fourier transform but based on a wide variety of functions. Each of these “mother wavelet” functions possesses specific properties that make it useful for understanding various interesting features of a waveform’s structure, not only its periodicities.⁵³ There are an infinite number of possible wavelets, and given a particular signal to analyze and purpose for analyzing it, some of those wavelets will perform better (i.e., more efficiently and accurately) than others.⁵⁴ Indeed, it may be the case that different wavelets are better suited to handling different parts of a waveform — the different movements of a symphony in a single audio recording, for example. The idea that an integral transform could be derived from any given source domain concept (perhaps through the creation of a custom wavelet based on the source function) overcomes the first problem with conceiving metaphors as transforms. The second problem can be resolved by composing the integral transform with its inverse transform, a mapping which recreates (an approximation of) the original function from its frequency spectrum.⁵⁵ The result will be a new function that approximates the analyzed function, but possibly with some aspects of the analyzing function incorporated. Finally, an updated, refined version of the original Lakovian metametaphor has been reached: METAPHOR IS A TRANSFORM-INVERSE TRANSFORM COMPOSITION MAPPING.

This TRANSFORM metametaphor has several promising features to recommend it. First, insofar as it is an elaboration of the Lakovian metametaphor METAPHORS ARE MATHEMATICAL MAPPINGS, it inherits the positive features of its ancestral mapping recounted earlier in this chapter. In particular, the TRANSFORM metametaphor does a good job of capturing the

⁵³Barbara Burke Hubbard, *The World According to Wavelets* (Wellesley: AK Peters, 1998), 32–3. As even the most basic explanation of wavelet analysis would be both relatively inaccessible and unnecessary for the purposes of this metaphorical discussion, I refer the interested reader to the excellent resource cited here, a refreshing diptych of mathematical precision and rigor juxtaposed with widely accessible sections discussing the historical development and practical repercussions of wavelet analysis.

⁵⁴Hubbard 239.

⁵⁵A composition of a transform with its inverse is one way that noise filtering can be accomplished: transform a waveform into its frequency spectrum (that is, *analyze* it), eliminate frequencies associated with noise from the spectrum (usually by discarding coefficients smaller than some given threshold), then rebuild the waveform from the modified frequency spectrum using the inverse transform(*synthesize* it).

asymmetry of metaphor: the roles of the analyzing function and the analyzed function in a transform are significantly different, and this difference is exhibited in the output of the transform. There is also some reason to believe that the TRANSFORM metametaphor could cohere with a neurological understanding of metaphor like the one Lakoff now espouses. The fact that neuroscientists interpret neural oscillations — often called brain *waves* — as symptoms of brain activity of various kinds seems coherent with the idea that concepts are waveforms. Moreover, there is empirical evidence that our perceptive pathways perform Fourier analysis; it thus seems reasonable that metaphor and other cognitive mechanisms could involve transforms as well.⁵⁶ However, the main advantage of METAPHOR IS A TRANSFORM-INVERSE TRANSFORM COMPOSITION MAPPING is arguably that it can explain the creation of target domain entities, since the inability of METAPHORS ARE MATHEMATICAL MAPPINGS to do so led to its retirement. Recall that many abstract concepts — LOVE, for example — possess minimal inherent structure so inherit much of their structure via conceptual metaphors.⁵⁷ Since a mathematical mapping cannot add new elements to its codomain but only specifies connexions between two preexisting sets, a metaphor cannot create target entities insofar as the target domain of the metaphorical mapping is understood as its codomain. Under the proposed metametaphor, however, a target concept waveform is transformed into a new waveform that is seen as a modified version of the original concept; taking the difference of the two waveforms exposes the structures that have been created or modified by the mapping. In particular, consider a target which is either a waveform that has gaps in it or which is merely a discrete set of points. *Interpolation* involves finding a curve that connects the given datapoints of the target, replacing gappy data with a continuous function; the act of interpolating can be understood as creating the entities that fill the gaps. While there are a variety of approaches to interpolation, wavelets provide one way of doing the job.⁵⁸ Thus, the composition of transforms provides a way of seeing a target concept in terms of another even if the target is sparse.

A waveform and its transform are two sides of the same coin, representations of the same object that emphasize different aspects. This relationship provides an opportunity

⁵⁶See De Valois, De Valois, and Yund (1978) for details on Fourier analysis in visual perception. Wavelets have also been hypothesized as playing a role in visual and auditory perception (Hubbard 69–70).

⁵⁷Lakoff and Johnson, *Philosophy in the Flesh* 70–1.

⁵⁸Yves Nievergelt, *Wavelets Made Easy* (Boston: Birkhäuser, 1999), 82–3.

to metaphorically connect CMT to other promising contemporary theories of concepts and metaphor discussed in chapter 3. Jesse Prinz’s theory of concepts as proxytypes builds upon Lawrence Barsalou’s claim that “concepts cannot be identified with the totality of category knowledge stored in long-term memory.”⁵⁹ Prinz views concepts as “mental representations of categories that are or *can be* activated in working memory.”⁶⁰ When a concept is required, its instantiation in working memory is shaped by the context of activation. The TRANSFORM metametaphor provides a potentially useful way of understanding proxytypes. Knowledge networks stored in long-term memory could be interpreted as frequency spectra. From time to time, a circumstance arises — a perception, a memory, a thought — that requires representation, “highlighting” some components of the frequency spectrum.⁶¹ A proxytype would then be activated in working memory by applying an inverse transform to the highlighted spectrum components. Conversely, active thoughts and experiences could modify our dormant knowledge networks through an application of the transform. This is an appealing view insofar as it mirrors the way media files on a computer are handled. A visual or sonic waveform is transformed and stored as frequency coefficients; it is typically more efficient to encode a waveform as its frequency spectrum than to represent it directly. When a user wants to access the media in the file, the computer decodes the information by applying the inverse transform to recreate the waveform. One question that needs to be addressed is what inverse transform is being applied in the activation of a proxytype? Whereas the source domain concept that defines the transform is specified in the case of metaphor, no such indication is made in this case. One possible way around this problem is to posit the existence of a standard, default conceptual transform; this would have the advantage of providing one way of differentiating between conceptual metaphor and other cognitive mappings. These metaphorical elaborations on the nature of concepts may also provide resources for conceptualizing some of Cornelia Müller’s important distinctions. A novel metaphor could be interpreted as having an analyzing wavelet that is still under development, an entrenched metaphor would have a well-established transform, and a dead metaphor’s transform would

⁵⁹Prinz 148.

⁶⁰Prinz 149; emphasis his.

⁶¹This highlighting would probably involve a combination of interaction between frequency functions with some thresholding.

no longer exist, the analyzing function having been lost. Additionally, the sleeping/waking dimension of Müller’s theory could correspond with the amount of thresholding that occurs in the composite mapping process and therefore with the amount of the analyzing function that shows through in the result.⁶² These metaphorical sketches are a bit rough, but do provide further support for the idea that CMT could be fruitfully integrated with other promising theories.

Mathematical metametaphors provide a useful way of understanding metaphor. The function-space and transform-operator metaphors introduced in this chapter not only rejuvenate and improve upon Lakoff and Johnson’s original metametaphorical understanding of conceptual metaphor, but also provide potential connexions to both their own contemporary neurological understanding as well as some noteworthy alternative philosophical theories of concepts and metaphor. The metaphorical work that has been done here is promising, but also very preliminary and rough; the implications of this approach are far from fully realized. In particular, very little has been said about the disanalogous aspects of the TRANSFORM metametaphor — and there are likely to be many. Readers interested in developing this metametaphor further may wish to first focus on the CONCEPTS ARE FUNCTIONS part of the metaphor; in particular, very little was said about how to understand intercategory structure within the conceptual function space. Readers interested in the possibility of developing this approach less metaphorically and more rigorously should seek out the disanalogies and consider whether they can be circumvented. There are certainly other mathematical metametaphors that could be considered; the specific approach taken here is primarily a function of my specific mathematical background, which featured undergraduate research on wavelet analysis and graduate work studying aperiodic tilings using mathematical diffraction (a phenomenon understood in terms of Fourier transforms). Finally, to those readers who have somehow made it to the end of the chapter despite finding the approach taken in this chapter bewildering and distasteful, I encourage you to consider this writing not as sloppy mathematics but more akin to strange (and probably bad) poetry: it does aim at the truth, but it uses a shotgun.

⁶²Müller 11.

CHAPTER 6

CONCLUSION

Hofstadter's Law: It always takes longer than you think it will take,
even when you take into account Hofstadter's Law.¹
— Douglas Hofstadter

Endings are often difficult, and such is the case here. This project has been an integral part of my existence for nearly a third of my life, and, as it comes to a close, I feel as though I am losing an important part of myself. Furthermore, the project feels incomplete insofar as that there is so much more to be said about mathematics and metaphor. However, it is more healthy — and more apt — to conceive of the denouement of my dissertation as metamorphic rather than injurious or abortive. To paraphrase a pearl of wisdom imparted to me by several individuals over the last few years, a doctorate is meant to be the beginning of a scholastic career, not the culmination of one.² That is, though the current project must end without being fully definitive and conclusive, research on this topic can (and should) continue for years to come. Thus, to conclude, I briefly review what this dissertation has accomplished, and indicate some prospects for future research and application.

It seems impossible to provide a comprehensive survey of metaphor scholarship in a single volume. As Wayne Booth noted in 1977, “[e]xplicit discussions of something called metaphor have multiplied astronomically in the past fifty years. . . students of metaphor have positively pullulated,” an observation which remains relevant over thirty years later.³ The best anyone can hope to do is summarize the important highlights. Given that other authors (such as Johnson and Leezenberg) have provided good synopses of metaphor elsewhere that

¹Hofstadter, *I am a Strange Loop* (New York: Basic Books, 2007), *xiv*.

²It can be somewhat difficult to remember this within the current economic climate, where academic positions are scarce and the competition is fierce.

³Wayne C. Booth, “Metaphor as Rhetoric: The Problem of Evaluation,” *On Metaphor*, Ed. Sheldon Sacks (Chicago: U Chicago P, 1978), 47.

I have made reference to, what does my overview contribute? First, unlike the synopses of other authors, mine is not *elsewhere*: it provides the reader with a convenient and easily accessible resource for obtaining the understanding of metaphor necessary for comprehending the rest of the dissertation. And second, while there is necessarily considerable overlap between the synopsis provided here and those of other authors, there are also some noteworthy differences. In particular, my précis of metaphor in chapters 2 and 3 emphasizes the history and development of conceptual metaphor theories. Several scholars — both historical and contemporary — connected to that tradition that are downplayed or overlooked in other synopses receive attention here, including Vico, Nietzsche, Ricoeur, Eco, and Müller. Though the contribution to the overall project is arguably minimal, it is noteworthy that I have never encountered discussion of Nicomachus in any other writings on metaphor. One obvious direction for future research would be to expand and improve upon the literature review of metaphor scholarship started here.

This dissertation argues that metaphor is conceptual in nature, but does not offer a novel theory of conceptual metaphor. Instead, Lakoff's CMT is defended as a plausible — albeit flawed — version of a theory of metaphor as conceptual. While many others have defended the Lakovian position against critics, the defense provided here differs from the others in a few key ways. First, supporters and critics of CMT alike tend to focus their attention on one or two major works in the Lakovian oeuvre. The synopsis of CMT I provide in chapter 3 is more comprehensive than most, making explicit reference to more than a dozen books and papers published by Lakoff, Johnson, and their collaborators. This thorough approach overcomes criticisms dependent upon either an oversimplified understanding of CMT or the equivocal importation of traditional linguistic assumptions. Second, my defense of conceptual metaphor is structured by CONCEPT, categorizing the various strengths and weaknesses of CMT. This approach emphasizes the fact that any theory of metaphor as conceptual must be affiliated with an account of concepts, and it exposes commonalities between CMT and other conceptual approaches that future authors might exploit to create a novel robust hybrid. Ultimately, though I think that there is much to be said in favour of the Lakovian approach, I am not wholeheartedly committed to the specifics of CMT but only to a conceptual approach to metaphor in general. Given the centrality of the notion

CONCEPT to philosophy and cognitive science, and given the current amount of disagreement on the topic, it seems there will be a need for scholarship in this area for years to come.

The overview of metaphor provided in chapters 2 and 3 is a necessary and interesting component of my project, but is ultimately peripheral: the heart of this dissertation is the argument that metaphor plays a constitutive role in mathematical practice. This general approach is motivated in part by the observation that the traditionally popular philosophical theories of mathematics each capture some important aspect of mathematics, but also possess fatal flaws that make them untenable. Adopting a framework that emphasizes conceptual metaphor provides a mechanism that allows for a partial reconciliation of these apparently incompatible theories of mathematics; chapter 4 begins this reconciliation process. Chapter 4 also considers several specific contemporary theories of mathematics amenable to the idea of constitutive mathematical metaphor, and claims that coherences exist between them that could allow them to collaboratively support each other rather than competing. In particular, the suggestion that Lakovian embodied mathematics and Yablonian figuralism are compatible seems obvious yet has not been discussed in the literature before now. Thus, like chapter 3, chapter 4 does not provide a novel theory but rather argues in favour of the development of a complex hybrid theory of mathematics with conceptual metaphor at its core that incorporates the strengths of a variety of theories usually viewed as rivals. Some important preliminary groundwork has been provided here, but the realization of such a hybrid theory will require significantly more work; this is an obvious direction for future research.

Chapter 5 argues that, even if the arguments of chapter 4 are airtight, it is nonetheless useful and non-question-begging to understand metaphor in terms of mathematics. Rather than looking to develop a rigorous computational model of metaphor, I instead sketch a novel metametaphor mapping the mathematical source domain of WAVELET TRANSFORMS to the target domain METAPHORS. The brevity and speculative nature of this chapter presents many opportunities for future research. First, the paltry history of computational linguistics provided could be significantly expanded and the state of the art of cognitive and linguistic modeling could be surveyed. Second, the TRANSFORM metametaphor sketched in chapter 5 could be further developed and refined. Third, additional novel mathematical metametaphors could be developed that help enrich our understanding of metaphor while simultaneously

lending support to this somewhat unorthodox and taboo use of mathematical concepts. And, fourth, the possibility of adapting the metametaphor given in chapter 5 into a rigorous mathematical model of metaphor could be seriously entertained. Such an inquiry might lead to the development of such a model, or to an argument explaining why such a model is not feasible (or outright impossible). Either of these outcomes would be rewarding.

The repercussions of my research are primarily theoretical rather than practical. Mathematics is clearly a highly productive and successful endeavour as it currently stands, and my research aims to describe and explain mathematical practices, not to alter them. The impact of my research, if any, will therefore be felt in the philosophy of mathematics, not within mathematics itself. If there is one area where I might hope my research could be applied, it is in mathematics education. Whereas mathematics research is highly successful as it stands, many people learn to dislike and avoid mathematics. While not every person needs to be a professional mathematician, the combination of increasing the average level of mathematical competency while simultaneously decreasing the amount of animosity towards mathematics could only be beneficial in this increasingly scientific age. Insofar as metaphor plays a constitutive role in mathematics, and insofar as metaphor is one of the most effective mechanisms humans have for comprehending the unknown, it seems that strategically augmenting the existing mathematics curriculum with metaphorical explanations may be an effective way of improving mathematics education. Though it clearly does not constitute a controlled scientific study, over the last decade I have noticed an improvement in the uptake of my mathematics pupils that correlates with my becoming more mindful of using apt metaphors in conjunction with rigorous definitions and proofs. I am therefore convinced that such an approach has merit, though it is clear that more rigorous evidence of its efficacy is desirable.

REFERENCES

- Aristotle. *Poetics*. Trans. I. Bywater. *The Complete Works of Aristotle*. Vol. 2. Ed. Jonathan Barnes. Princeton: Princeton UP, 1984. 2316–40. Print.
- . *Rhetoric*. Trans. J.H. Freese. *Aristotle in 23 Volumes*. Vol. 22. Cambridge: Harvard UP, 1926. *Perseus Digital Library*. Tufts U. Web. 22 Feb. 2009.
- . *Rhetoric*. Trans. W. Rhys Roberts. *The Complete Works of Aristotle*. Vol. 2. Ed. Jonathan Barnes. Princeton: Princeton UP, 1984. 2152–269. Print.
- Athenaeus. *The Learned Banqueters*. Vol. 3. Ed. and trans. Douglas Olson. Cambridge: Harvard UP, 2008. Print.
- Austin, John L. *How to Do Things With Words*. Ed. J.O. Urmson and Marina Sbisa. 2nd ed. Oxford: Clarendon, 1975. Print.
- Balaguer, Mark. “Fictionalism in the Philosophy of Mathematics.” *Stanford Encyclopedia of Philosophy*. 16 Sept. 2011. Web. 30 Jul. 2012.
- Barrow, John D. *Pi in the Sky: Counting, Thinking, and Being*. Oxford: Clarendon, 1992. Print.
- Barsalou, Lawrence W., et al. “Concepts and Meaning.” *Chicago Linguistics Society 29: Papers from the parasession on conceptual representations*. Ed. K. Beals et al. Chicago: Chicago Linguistics Society, 1993. 1–41. *Lawrence Barsalou Website*. Emory University. Web. 26 Nov. 2011.
- Bateson, Gregory. *Mind and Nature*. New York: E.P. Dutton, 1979. Print.
- Benacerraf, Paul. “Mathematical Truth.” *The Journal of Philosophy* 70.19 (1973): 661–79. *JSTOR*. Web. 17 May 2012.
- Benacerraf, Paul and Hilary Putnam, eds. *Philosophy of Mathematics: Selected Readings*. Englewood Cliffs: Prentice-Hall, 1964. Print.
- Bhatia, Rajendra. *Fourier Series*. Washington, DC: Mathematical Association of America, 2005. Print.
- Black, Max. “Metaphor.” *Proc. of Aristotelian Society* 55 (1954–55): 273–94. Reprinted in Johnson, *Philosophical Perspectives on Metaphor* 63–82. Print.
- . *Models and Metaphors*. Ithaca: Cornell UP, 1962. Print.
- . “More About Metaphor.” 19–41. Ortony, *Metaphor and Thought* 19–41. Print.
- . “Review of *Metaphors We Live By*.” *J. of Aesthetics and Art Criticism* 40.2 (1981): 208–10. *JSTOR*. Web. 8 May 2009.
- Bloor, David. *Knowledge and Social Imagery*. 2nd ed. Chicago: U of Chicago P, 1991. Print.
- Bolzano, Bernard. *Contributions to a Better-Grounded Presentation of Mathematics*. Prague: Kaspar Widtmann, 1810. Found in *The Mathematical Works of Bernard Bolzano*. Ed. and Trans. Steve Russ. New York: Oxford UP, 2004. 83–137. *MyiLibrary*. Web. 31 Jan. 2012.
- Booth, Wayne C. “Metaphor as Rhetoric: The Problem of Evaluation.” *On Metaphor*. Ed. Sheldon Sacks. Chicago: U Chicago P, 1979. 47–70. Print.

- Boroditsky, Lera. "Metaphoric structuring: understanding time through spatial metaphors." *Cognition* 75.1 (2000): 1–28. *ScienceDirect*. Web. 2 Oct. 2013.
- Brouwer, L.E.J. *Brouwer's Cambridge Lectures on Intuitionism*. Ed. D. van Dalen. Cambridge: Cambridge UP, 1981. Print.
- Brown, James Robert. *Philosophy of Mathematics*. 2nd ed. New York: Routledge, 2008. Print.
- Burgess, John P. "Mathematics and *Bleak House*." *Philosophia Mathematica* 12.1 (2004): 18–36. *EBSCOhost*. Web. 15 Apr. 2013.
- Camp, Elisabeth. "Two Varieties of Literary Imagination: Metaphor, Fiction, and Thought Experiments." *Midwest Studies in Philosophy*. Ed. Peter A. French, Howard K. Wettstein, and Ernie Lepore. Vol. 33. Boston: Wiley-Blackwell, 2009. 107–30. Print.
- Camp, Elisabeth and Marga Reimer. "Metaphor." *Handbook of Philosophy of Language*. Ed. Lepore and Smith. Oxford: Oxford UP, 2006. 845–63. Print.
- Carazo, Pau, et al. "Quantity discrimination in *Tenebrio molitor*: evidence of numerosity discrimination in an invertebrate?" *Animal Cognition* 12.3 (2009): 463–70. *SpringerLink*. 1 Jan. 2009. Web. 11 Aug. 2012.
- Carlyle, Thomas. *Past and Present*. Ed. Chris R. Vanden Bossche. Berkeley: U of California P, 2005. Print.
- Chalmers, David, Robert French, and Douglas Hofstadter. "High-level Perception, Representation, and Analogy: A Critique of Artificial-intelligence Methodology." *Fluid Concepts and Creative Analogies*. New York: Basic Books, 1995. 169–93. Print.
- "concept, n." *OED Online*. Mar. 2014. Oxford UP. Web. 25 Apr. 2014.
- "concipiō." *Oxford Latin Dictionary*. 1968. Print.
- Costelloe, Timothy. "Giambattista Vico." *Stanford Encyclopedia of Philosophy*. 15 Jul. 2008. Web. 11 Mar. 2009.
- Dabbs, James M. and Joseph P. Moorer. "Core Body Temperature and Social Arousal." *Personality and Social Psychology Bulletin* 1.3 (1975): 517–20. *SAGE Journals Online*. Web. 4 Jun. 2010.
- Dacke, Marie and Madyam V. Srinivasan. "Evidence for counting in insects." *Animal Cognition* 11.4 (2008): 683–89. *SpringerLink*. 27 May 2008. Web. 22 Jun. 2011.
- Danesi, Marcel. "A conceptual metaphor framework for the teaching of mathematics." *Studies in Philosophy and Education* 26.3 (2007): 225–36. *EBSCOhost*. Web. 8 Mar. 2009.
- Davidson, Donald. "A Nice Derangement of Epitaphs." *Truth, Language, and History*. New York: Oxford UP, 2005. 89–107. Print.
- . "Locating Literary Language." *Truth, Language, and History*. New York: Oxford UP, 2005. 167–81. Print.
- . "Reply to Oliver Scholz." *Reflecting Davidson: Donald Davidson Responding to an International Forum of Philosophers*. Ed. Rolf Stoecker. New York: de Gruyter, 1993. 172–73. Print.
- . "Truth and Meaning." *Synthese* 17 (1967): 304–23. Reprinted in Martinich, *The Philosophy of Language* 98–109. Print.
- . "What Metaphors Mean." *On Metaphor*. Ed. Sheldon Sacks. Chicago: U of Chicago P, 1979. 29–46. Reprinted in Martinich, *The Philosophy of Language* 435–46. Print.
- Dawson, John W. *Logical Dilemmas: The Life and Work of Kurt Gödel*. Wellesley, MA: AK Peters, 1997. Print.

- de Bruijn, N.G. “Algebraic theory of Penrose’s non-periodic tilings of the plane I.” *Indagationes Mathematicae* 84.1 (1981): 39–52. *ScienceDirect*. Web. 23 Oct. 2013.
- Dehaene, Stanislas. *The Number Sense: How the Mind Creates Mathematics*. 2nd ed. New York: Oxford UP, 2012. Print.
- Descartes, René. *Discourse on Method*. Trans. Donald Cress. 3rd ed. Indianapolis: Hackett, 1998. Selections reprinted in *René Descartes: Philosophical Essays and Correspondence*, Ed. Roger Ariew, (Indianapolis: Hackett, 2000), 46–82. Print.
- Detlefsen, Michael. “On an alleged refutation of Hilbert’s program using Gödel’s first incompleteness theorem.” *Journal of Philosophical Logic* 19.4 (1990): 343–77. *JSTOR*. Web. 7 May 2014.
- Deutscher, Guy. *The Unfolding of Language*. New York: Metropolitan, 2005. Print.
- De Valois, Karen, Russell De Valois, and E. William Yund. “Responses of Striate Cortex Cells to Grating and Checkerboard Patterns.” *Journal of Physiology* 291 (1979): 483–505. *PubMed*. Web. 3 May 2010.
- Devlin, Keith. *Mathematics: The Science of Patterns*. New York: Scientific American Library, 1997. Print.
- du Sautoy, Marcus. “Joseph Fourier.” *A Brief History of Mathematics*. BBC Radio 4. Podcast audio. 16 Jun. 2010. MP3. 2 Feb. 2012.
- Eco, Umberto. “Metaphor, Dictionary, and Encyclopedia.” *New Literary History* 15.2 (1984): 255–71. *JSTOR*. Web. 12 Jan. 2012.
- . *Semiotics and the Philosophy of Language*. Bloomington: Indiana UP, 1984. Print.
- Ernest, Paul. *Social Constructivism as a Philosophy of Mathematics*. Albany: SUNY Press, 1998. Print.
- Falkenhainer, Brian, Kenneth D. Forbus, and Dedre Gentner. “The Structure-Mapping Engine: Algorithm and Examples.” *Artificial Intelligence* 41 (1989): 1–63. *Dedre Gentner’s Web Page*. Northwestern University. Web. 13 Aug. 2013.
- Fauconnier, Gilles. *Mappings in Thought and Language*. New York: Cambridge UP, 1997. Print.
- Fauconnier, Gilles and Mark Turner. *The Way We Think: Conceptual Blending and the Mind’s Hidden Complexities*. New York: Basic Books, 2002. Print.
- Field, Hartry. *Realism, Mathematics and Modality*. Oxford: Blackwell, 1989. Print.
- . *Science Without Numbers*. Princeton: Princeton UP, 1980. Print.
- Fodor, Jerry A. *Concepts: Where Cognitive Science Went Wrong*. New York: Oxford UP, 1998. Print.
- Freeman, Kathleen. *Ancilla to the Pre-Socratic Philosophers*. Cambridge: Harvard UP, 1948. Print.
- Frege, Gottlob. *The Foundations of Arithmetic*. Trans. J.L. Austin. 2nd ed. New York: Harper and Brothers, 1953. Print.
- Gallese, Vittorio and George Lakoff. “The Brain’s Concepts.” *Cognitive Neuropsychology* 22.3 (2005): 455–79. *EBSCOhost*. Web. 15 Mar. 2010.
- Gentner, Dedre. “Structure Mapping: A Theoretical Framework for Analogy.” *Cognitive Science* 7 (1983): 155–70. *Dedre Gentner’s Web Page*. Northwestern University. Web. 17 Feb. 2010.

- Gentner, Dedre, et al. "Metaphor Is Like Analogy." *The Analogical Mind: Perspectives from Cognitive Science*. Ed. Dedre Gentner, Keith J. Holyoak, and Boicho N. Kokinov. Cambridge: MIT Press, 2001. 199–253. Print.
- Gibbs, Raymond W. "Evaluating Conceptual Metaphor Theory." *Discourse Processes* 48.8 (2011): 529–62. *Taylor and Francis Online*. Web. 1 Oct. 2013.
- , ed. *The Cambridge Handbook of Metaphor and Thought*. New York: Cambridge UP, 2008. Print.
- . *The Poetics of Mind*. New York: Cambridge UP, 1994. Print.
- . "Why many concepts are metaphorical." *Cognition* 61 (1996): 309–19. *ScienceDirect*. Web. 23 Apr. 2006.
- Gold, Bonnie. "Read This! The MAA Online book review column: Where Mathematics Comes From: How the Embodied Mind Brings Mathematics Into Being." *MathDL*. The Mathematical Assoc. of America, 3 Feb. 2001. Web. 5 May 2009.
- Goldin, Gerald A. "Review: Counting on the Metaphorical." *Nature* 413.6851 (2001): 18–19. *EBSCOhost*. Web. 22 Aug. 2007.
- Gómez-Laplaza, Luis M. and Robert Gerlai. "Spontaneous discrimination of small quantities: shoaling preferences in angelfish (*Pterophyllum scalare*)." *Animal Cognition* 14.4 (2011): 565–74. *SpringerLink*. 16 Mar. 2011. Web. 22 Jun. 2011.
- Grady, Joseph. "Foundations of Meaning: Primary Metaphors and Primary Scenes." Diss. Berkeley, 1997. Print.
- Grady, Joseph E., Todd Oakley, and Seana Coulson. "Blending and metaphor." *Metaphor in Cognitive Linguistics*. Ed. Raymond W. Gibbs and Gerard J. Steen. Amsterdam and Philadelphia: John Benjamins, 1999. 101–24. Print.
- Grattan-Guinness, Ivor. *The Norton History of the Mathematical Sciences*. New York: W.W. Norton and Company, 1997. Print.
- Grice, H.P. "Logic and Conversation." *Syntax and Semantics*. Ed. Peter Cole and Jerry L. Morgan. Vol. 3. New York: Academic P, 1975. 41–58. Reprinted in Martinich, *The Philosophy of Language* 165–175. Print.
- . "Meaning." *Philosophical Review* 66 (1957): 377–88. Reprinted in Martinich, *The Philosophy of Language* 92–97. Print.
- Guhe, Markus, Alan Smaill, and Alison Pease. "A Formal Cognitive Model of Mathematical Metaphors." *KI 2009: Advances in Artificial Intelligence. 32nd Annual German Conference on AI, Paderborn, Germany, September 15-18, 2009*. Ed. B. Mertsching, M Hund, and Z Aziz. Berlin: Springer, 2009. 323–30. *Alison Pease's Web Page*. University of Edinburgh School of Informatics. Web. 4 Feb. 2013.
- Hilbert, David. *David Hilbert's Lectures on the Foundations of Arithmetic and Logic 1917–1933*. Ed. William Ewald and Wifried Sieg. Heidelberg: Springer-Verlag, 2013. Print.
- . "Mathematical Problems." *Bulletin of the American Mathematical Society* 8.10 (1902): 437–79. *Project Euclid*. Web. 7 May 2014.
- . "On the Infinite." Trans. Erna Putnam and Gerald J. Massey. Benacerraf and Putnam. 134–51. Originally published in *Mathematische Annalen* 95 (1925): 161–90. Print.
- Hinks, D.A.G. "Tisias and Corax and the Invention of Rhetoric." *Classical Quarterly* 34.1 (1940): 61–9. *JSTOR*. Web. 25 Apr. 2014.
- Hobbes, Thomas. *Leviathan*. Ed. Edwin Curley. Indianapolis: Hackett, 1994. Print.

- Hoffman, Sarah. "Mathematics as Make-Believe: A Constructive Empiricist Account." Diss. U of Alberta, 1999. Print.
- . "Yablo's Figuralist Account of Mathematics." (2012): 1–20. Unpublished Article.
- Hofstadter, Douglas R. "Analogies and Metaphors to Explain Gödel's Theorem." *The Two-Year College Mathematics Journal* 13.2 (1982): 98–114. *JSTOR*. Web. 18 Feb. 2013.
- . "Epilogue: Analogy as the Core of Cognition." *The Analogical Mind: Perspectives from Cognitive Science*. Ed. Dedre Gentner, Keith J. Holyoak, and Boicho N. Kokinov. Cambridge: MIT Press, 2001. 499–538. Print.
- . *I am a Strange Loop*. New York: Basic Books, 2007. Print.
- Hofstadter, Douglas R. and the Fluid Analogies Research Group. *Fluid Concepts and Creative Analogies*. New York: Basic Books, 1995. Print.
- Holyoak, Keith J. "Overview of Research Career." *Keith Holyoak's Web Page*. UCLA Reasoning Lab. 31 Jul. 2012. Web. 18 Aug. 2013.
- Holyoak, Keith J., Dedre Gentner, and Boicho N. Kokinov. "Introduction: The Place of Analogy in Cognition." *The Analogical Mind: Perspectives from Cognitive Science*. Ed. Dedre Gentner, Keith J. Holyoak, and Boicho N. Kokinov. Cambridge: MIT Press, 2001. 1–20. Print.
- Holyoak, Keith J. and Paul Thagard. "Analogical Mapping by Constraint Satisfaction." *Cognitive Science* 13 (1989): 295–355. *Keith Holyoak's Web Page*. UCLA Reasoning Lab. 31 Jul. 2012. Web. 18 Aug. 2013.
- Horsten, Leon. "Philosophy of Mathematics." *Stanford Encyclopedia of Philosophy*. 2 May 2012. Web. 7 May 2012.
- Hubbard, Barbara Burke. *The World According to Wavelets: The Story of a Mathematical Technique in the Making*. 2nd ed. Wellesley: AK, 1998. Print.
- "Inside Google Translate." *Google Translate*. Google, n.d. Web. 28 Jan. 2014.
- Islamic Art and Geometric Design*. New York: Museum of Modern Art, 2004. *Google Books*. Web. 8 July 2013.
- Isocrates. *Cyprian Orations*. Ed. Edward Forster. Oxford: Clarendon, 1912. *Internet Archive*. Web. 25 Apr. 2014.
- Jackendoff, Ray and David Aaron. "Review of *More Than Cool Reason*." *Language* 67.2 (1991): 320–38. *JSTOR*. Web. 8 May 2009.
- Johnson, Christopher. "Metaphor vs. conflation in the acquisition of polysemy: The case of see." *Cultural, Psychological, and Typological Issues in Cognitive Linguistics*. Ed. Masako Hiraga, Chris Sinha, and Sherman Wilcox. Philadelphia: John Benjamins, 1999. 155–70. Print.
- Johnson, Mark. "Introduction: Metaphor in the Philosophical Tradition." M. Johnson, *Philosophical Perspectives on Metaphor* 3–47. Print.
- , ed. *Philosophical Perspectives on Metaphor*. Minneapolis: U of Minnesota P, 1981. Print.
- . *The Body in the Mind*. Chicago: U of Chicago P, 1986. Print.
- Johnson, Mark and George Lakoff. "Why cognitive linguistics requires embodied realism." *Cognitive Linguistics* 13.3 (2002): 245–63. *Gale Cengage*. Web. 7 Nov. 2011.
- Kirby, John T. "Aristotle on Metaphor." *American Journal of Philology* 118 (1997): 517–54. *JSTOR*. Web. 6 Jan. 2008.
- Kitcher, Philip. *The Nature of Mathematical Knowledge*. New York: Oxford UP, 1984. Print.

- Kline, Morris. *Mathematical thought from ancient to modern times*. New York: Oxford UP, 1972. Print.
- Kövecses, Zoltán. *Metaphor: A Practical Introduction*. New York: Oxford UP, 2002. Print.
- Kusch, Martin. "Psychologism." *Stanford Encyclopedia of Philosophy*. 7 Nov. 2011. Web. 14 Apr. 2012.
- Lakatos, Imre. *Proofs and Refutations*. Ed. John Worrall and Elie Zahar. New York: Cambridge UP, 1976. Print.
- Lakoff, George. "The Contemporary Theory of Metaphor." 202–51. Ortony, *Metaphor and Thought* 202–51. Print.
- . "The Death of Dead Metaphor." *Metaphor and Symbolic Activity* 2.2 (1987): 143–7. *Informaworld*. Web. 23 May 2009.
- . "The Meanings of Literal." *Metaphor and Symbolic Activity* 1.4 (1986): 291–6. *EBSCOhost*. Web. 20 Nov. 2011.
- . "The Neural Theory of Metaphor." 17–38. Gibbs, *The Cambridge Handbook of Metaphor and Thought* 17–38. Print.
- . *Women, Fire, and Dangerous Things*. Chicago: U of Chicago P, 1987. Print.
- Lakoff, George and Mark Johnson. *Metaphors We Live By*. 1980. With a new afterword. Chicago: U of Chicago P, 2003. Print.
- . *Philosophy in the Flesh*. New York: Basic Books, 1999. Print.
- Lakoff, George and Rafael E. Núñez. "Reply to Bonnie Gold's review of 'Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being'." *MathDL*. The Mathematical Assoc. of America, 3 Feb. 2001. Web. 5 May 2009.
- . "The Metaphorical Structure of Mathematics: Sketching Out Cognitive Foundations for a Mind-Based Mathematics." *Mathematical Reasoning: Analogies, Metaphors, and Images*. Ed. L. English. Hillsdale, NJ: Erlbaum, 1997. 21–89. Print.
- . *Where Mathematics Comes From*. New York: Basic Books, 2000. Print.
- Lakoff, George and Mark Turner. *More Than Cool Reason*. Chicago: U of Chicago P, 1989. Print.
- Langacker, Ronald W. "An Introduction to Cognitive Grammar." *Cognitive Science* 10.1 (1986): 1–40. *Elsevier ScienceDirect*. Web. 29 Oct. 2011.
- Laurence, Stephen and Eric Margolis. "Concepts and Cognitive Science." *Concepts: Core Readings*. Ed. Eric Margolis and Stephen Laurence. Cambridge: MIT Press, 1999. 3–81. Print.
- Leezenberg, Michiel. *Contexts of Metaphor*. Amsterdam: Elsevier, 2001. Print.
- Leibniz, Gottfried. "On the Universal Science: Characteristic (Parts XIV and XV)." *Monadology and Other Philosophical Essays*. Ed. and trans. Paul Schrecker and Anne Martin Schrecker. New York: Bobbs-Merrill, 1965. 11–21. Print.
- Liddell, George Henry and Robert Scott. *A Greek-English Lexicon*. 9th ed. Oxford: Clarendon, 1940. *Perseus Digital Library*. Tufts U. Web. 4 Feb. 2009.
- Linnebo, Øystein. "Platonism in the Philosophy of Mathematics." *Stanford Encyclopedia of Philosophy*. 25 Jul. 2011. Web. 2 May 2012.
- "literal, adj. and n." *OED Online*. Dec. 2011. Oxford UP. Web. 21 Feb. 2012.
- Locke, John. *An Essay Concerning Human Understanding*. Ed. Peter H. Nidditch. Oxford: Clarendon, 1975. Print.

- Lu, Hongjing, Dawn Chen, and Keith J. Holyoak. "Bayesian Analogy with Relational Transformations." *Psychological Review* 119.3 (2012): 617–48. *Keith Holyoak's Web Page*. UCLA Reasoning Lab. Web. 18 Aug. 2013.
- Mac Cormac, Earl R. *A Cognitive Theory of Metaphor*. Cambridge: Bradford, 1985. Print.
- Mac Lane, Saunders. *Mathematics: Form and Function*. New York: Springer-Verlag, 1986. Print.
- Manin, Yuri Ivanovich. "Mathematics as Metaphor." *Proceedings of the International Congress of Mathematicians, August 21–29, 1990*. Ed. Ichiro Satake. Vol. 2. Tokyo: Springer-Verlag, 1991. 1665–72. *mathunion.org*. Web. 27 Jan. 2013.
- Martinich, A.P. "A Theory for Metaphor." *Journal of Literary Semantics* 13 (1984): 35–56. Reprinted in Martinich, *The Philosophy of Language* 447–458. Print.
- . "A Theory of Fiction." *Philosophy and Literature* 25 (2001): 96–112. *Project Muse*. Johns Hopkins UP. Web. 5 Oct. 2006.
- , ed. *The Philosophy of Language*. 4th ed. New York: Oxford UP, 2001. Print.
- "mathematic, n. and adj." *The Oxford English Dictionary*. 3rd ed. 2001. *OED Online*. Oxford UP. Web. 17 May 2012.
- McCall, Marsh H. *Ancient Rhetorical Theories of Simile and Comparison*. Cambridge: Harvard UP, 1969. Print.
- McGlone, Matthew S. "What is the explanatory value of a conceptual metaphor?" *Language and Communication* 27.2 (2007): 109–26. *Elsevier ScienceDirect*. Web. 29 Sept. 2008.
- "metaphor, n." *OED Online*. June 2008. Oxford UP. Web. 2 Feb. 2009.
- "metonymy, n." *OED Online*. Sept. 2011. Oxford UP. Web. 22 Sept. 2011.
- Müller, Cornelia. *Metaphors Dead and Alive, Sleeping and Waking*. Chicago: U of Chicago P, 2008. Print.
- Murphy, Gregory L. "On metaphoric representation." *Cognition* 60 (1996): 173–204. *ScienceDirect*. Web. 27 Sept. 2008.
- . *The Big Book of Concepts*. Cambridge: MIT Press, 2002. Print.
- Narayanan, Srinivas. "Embodiment in Language Understanding: Sensory-Motor Representations for Metaphoric Reasoning About Event Descriptions." Diss. Berkeley, 1997. Print.
- Nietzsche, Friedrich. "On Truth and Lying in an Extra-Moral Sense." *Friedrich Nietzsche on Rhetoric and Language*. Ed. Sander L. Gilman. Trans. David J. Parent. New York: Oxford UP, 1989. 246–57. Print.
- Nievergelt, Yves. *Wavelets Made Easy*. Boston: Birkhäuser, 1999. Print.
- Núñez, Rafael. "Conceptual Metaphor, Human Cognition, and the Nature of Mathematics." Gibbs, *The Cambridge Handbook of Metaphor and Thought* 339–62. Print.
- . "Numbers and arithmetic: neither hard-wired nor out there." *Biological Theory* 4.1 (2009): 68–83. *Núñez's personal website at UCSD*. Web. 9 Oct 2012.
- Núñez, Rafael, Kensey Cooperrider, and Jürg Wassmann. "Number concepts without number lines in an indigenous group of Papua New Guinea." *PLoS ONE* 7.4 (2012): 1–8. Web. 9 Oct. 2012.
- O'Connor, J.J. and E.F. Robertson. "Max Black." *MacTutor History of Mathematics Archive*. U of St. Andrews, Scotland., Mar. 2001. Web. 5 Mar. 2009.
- Ogden, C.K. and I.A. Richards. *The Meaning of Meaning*. 8th ed. New York: Harcourt, Brace, and Co., 1923. Print.

- Ortony, Andrew. "Metaphor: A multidisciplinary problem." *Metaphor and Thought*. Ed. Andrew Ortony. 1st ed. Cambridge: Cambridge UP, 1979. 1–18. Print.
- , ed. *Metaphor and Thought*. 2nd ed. New York: Cambridge UP, 1993. Print.
- Parsons, Glenn G. and James Robert Brown. "Platonism, Metaphor, and Mathematics." *Dialogue* 43.1 (2004): 47–66. Print.
- Peacock, Thomas Love. "The Four Ages of Poetry." *Peacock's Four Ages of Poetry, Shelley's Defence of Poetry, Browning's Essay on Shelley*. Ed. H.F.B Brett-Smith. Oxford: Blackwell, 1929. 3–19. Print.
- Pimm, David. "Mathematical Metaphor." *For the Learning of Mathematics* 8.1 (1988): 30–4. *JSTOR*. Web. 29 Jan. 2013.
- . "Metaphor and Analogy in Mathematics." *For the Learning of Mathematics* 1.3 (1981): 47–50. *JSTOR*. Web. 27 Jan. 2013.
- Plato. *Meno*. Trans. Benjamin Jowett. *The Internet Classics Archive*. Web Atomic and MIT. Web. 14 Apr. 2014.
- . *Phaedrus*. Trans. Harold N. Fowler. *Plato in Twelve Volumes*. Vol. 9. Cambridge: Harvard UP, 1925. *Perseus Digital Library*. Tufts U. Web. 18 Feb. 2009.
- . *Republic*. Trans. Benjamin Jowett. *The Internet Classics Archive*. Web Atomic and MIT, 7 Jan. 2009. Web. 6 Feb. 2009.
- . *Symposium*. Trans. Benjamin Jowett. *The Internet Classics Archive*. Web Atomic and MIT. Web. 8 Sept. 2009.
- . *Timaeus*. Trans. Benjamin Jowett. *The Internet Classics Archive*. Web Atomic and MIT. Web. 7 Feb. 2009.
- Poincaré, Henri. "The Future of Mathematics." *The Monist* 20.1 (1910): 76–92. *JSTOR*. Web. 26 Feb. 2012.
- Pólya, George. *Induction and Analogy in Mathematics*. Princeton: Princeton UP, 1954. Print.
- Postnikoff, Derek. "Mathematical metaphor: Linguistic or conceptual?" *18th Novembertagung on the History, Philosophy, and Didactics of Mathematics: Mathematical Practice and Development throughout History*. Ed. Ingo Witzke. Berlin: Logos Verlag, 2008. 3–14. Print.
- Presmeg, Norma. "Review: Mathematical Idea Analysis: A Science of Embodied Mathematics." *Journal for Research in Mathematics Education* 33.1 (2002): 59–63. *JSTOR*. Web. 6 Feb. 2008.
- Prinz, Jesse J. *Furnishing the Mind: Concepts and Their Perceptual Basis*. Cambridge: MIT Press, 2002. Print.
- Quine, W.V.O. "A Postscript on Metaphor." *On Metaphor*. Ed. Sheldon Sacks. Chicago: U Chicago P, 1979. 159–60. Print.
- Rakova, Marina. "The philosophy of embodied realism: A high price to pay?" *Cognitive Linguistics* 13.3 (2002): 215–44. *Gale Cengage*. Web. 7 Nov. 2011.
- Rapp, Christof. "Aristotle's Rhetoric." *Stanford Encyclopedia of Philosophy*. 2 May 2002. Web. 17 Feb. 2009.
- Reddy, Michael J. "The Conduit Metaphor - A Case of Frame Conflict in Our Language about Language." *Metaphor and Thought*. Ed. Andrew Ortony. 1st ed. Cambridge: Cambridge UP, 1979. 284–310. Print.
- Resnik, Michael D. *Frege and the Philosophy of Mathematics*. Ithaca: Cornell UP, 1980. Print.
- Richards, I.A. *The Philosophy of Rhetoric*. Oxford: Oxford UP, 1936. "Lecture V: Metaphor" reprinted in Johnson, *Philosophical Perspectives on Metaphor*, 48–62. Print.

- Ricoeur, Paul. *The Rule of Metaphor*. Trans. Robert Czerny. Toronto: U of Toronto P, 1975. Print.
- Rotman, Brian. "Gesture in the Head: Mathematics and Mobility." *Mathematics and Narrative conference, Mykonos, 2005. Thales + Friends*. Web. 19 Aug. 2009.
- . *Mathematics as Sign: Writing, Imagining, Counting*. Stanford: Stanford UP, 2000. Print.
- Russell, Bertrand. *The Philosophy of Logical Atomism*. London: Routledge, 2010. Print.
- Sawyer, W.W. *Prelude to Mathematics*. Harmondsworth, Middlesex: Penguin, 1955. Print.
- Schiralli, Martin and Nathalie Sinclair. "A Constructive Response to 'Where Mathematics Comes From'." *Educational Studies in Mathematics* 52.1 (2003): 79–91. *JSTOR*. Web. 25 Feb. 2007.
- Scholtz, Oliver. "'What Metaphors Mean' and how Metaphors Refer." *Reflecting Davidson*. Ed. Ralf Stoecker. New York: de Gruyter, 1993. 161–71. Print.
- Searle, John. "A Taxonomy of Illocutionary Actions." *Expression and Meaning*. Cambridge: Cambridge UP, 1979. 1–29. Print.
- . "Indirect Speech Acts." *Expression and Meaning*. Cambridge: Cambridge UP, 1979. 30–57. Print.
- . "Literal Meaning." *Expression and Meaning*. Cambridge: Cambridge UP, 1979. 117–36. Print.
- . "Metaphor." *Expression and Meaning*. Cambridge: Cambridge UP, 1979. 76–116. Print.
- . *Speech Acts*. Cambridge: Cambridge UP, 1969. Print.
- . "The Logical Status of Fictional Discourse." *Expression and Meaning*. Cambridge: Cambridge UP, 1979. 58–75. Print.
- Shakespeare, William. *Romeo and Juliet*. *Shakespeare Online*. Web. 24 Jul. 2013.
- Shapiro, Stewart, ed. *The Oxford Handbook of Philosophy of Mathematics and Logic*. Oxford: Oxford UP, 2005. Print.
- Shelley, Percy Bysshe. "A Defence of Poetry." *Peacock's Four Ages of Poetry, Shelley's Defence of Poetry, Browning's Essay on Shelley*. Ed. H.F.B Brett-Smith. Oxford: Blackwell, 1929. 23–59. Print.
- "simile, n." *The Oxford English Dictionary*. 2nd ed. 1989. *OED Online*. Oxford UP. Web. 18 Feb. 2009.
- Skorupski, John. "Later Empiricism and Logical Positivism." *The Oxford Handbook of Philosophy of Mathematics and Logic*. Ed. Stewart Shapiro. New York: Oxford UP, 2005. Print.
- Smith, David L. *Freud's Philosophy of the Unconscious*. Dordrecht: Kluwer, 1999. Print.
- Smith, Robin. "Aristotle's Logic." *Stanford Encyclopedia of Philosophy*. 23 Mar. 2011. Web. 28 Jan. 2014.
- Sutton, David. *Platonic and Archimedean Solids*. New York: Walker and Company, 2002. Print.
- "transfer, v." *OED Online*. Dec. 2011. Oxford UP. Web. 19 Feb. 2012.
- Triebel, Hans. *Fourier Analysis and Function Spaces*. Leipzig: Teubner, 1977. Print.
- Tsohatzidis, Savas L. "Speaker meaning, sentence meaning, and metaphor." *Foundations of Speech Act Theory: Philosophical and linguistic perspectives*. Ed. Savas L. Tsohatzidis. London: Routledge, 1994. Print.

- Turing, Alan M. "On Computable Numbers, with an Application to the Entscheidungsproblem." *Proceedings of the London Mathematical Society*. Series 2. 42 (1937): 230–65. *Oxford Journals Online*. Web. 13 Jun. 2012.
- Vico, Giambattista. *The New Science of Giambattista Vico*. Trans. Thomas Bergin and Max Fisch. New York: Doubleday Anchor, 1961. Print.
- Walton, Kendall. "Metaphor and Prop Oriented Make-Believe." *European Journal of Philosophy* 1.1 (1993): 39–56. Print.
- . *Mimesis as Make-Believe*. Cambridge: Harvard UP, 1990. Print.
- Wigner, Eugene. "The unreasonable effectiveness of mathematics in the natural sciences." *Communications on Pure and Applied Mathematics* 13.1 (1960): 1–14. Print.
- Wittgenstein, Ludwig. "A Lecture on Ethics." *The Philosophical Review* 74.1 (1965): 3–12. *JSTOR*. Web. 2 Mar. 2009.
- . *Philosophical Investigations*. Trans. G.E.M. Anscombe. 3rd ed. Oxford: Basil Blackwell, 1972. Print.
- . *Remarks on the Foundations of Mathematics*. Ed. G.H. von Wright, R. Rhees, and G.E.M. Anscombe. Trans. G.E.M. Anscombe. Cambridge: MIT Press, 1956. Print.
- Yablo, Stephen. "Abstract Objects: A Case Study." *Things*. New York: Oxford UP, 2010. 200–220. Print.
- . "Does Ontology Rest on a Mistake?" *Things*. New York: Oxford UP, 2010. 117–44. Print.
- . "Go Figure: A Path through Fictionalism." *Things*. New York: Oxford UP, 2010. 177–99. Print.
- . "Must Existence-Questions Have Answers?" *Things*. New York: Oxford UP, 2010. 296–314. Print.
- . "The Myth of the Seven." *Things*. New York: Oxford UP, 2010. 221–45. Print.
- . "Why *I* am Not a Nominalist." *Yablo's personal website*. 2003. Web. 9 Apr. 2013.
- Zwicky, Jan. "Mathematical Analogy and Metaphorical Insight." *The Mathematical Intelligencer* 28.2 (2006): 4–9. Print.